1. Find the duration of a perpetuity-immediate with payments of 1 every two years?

A. $\dfrac{1}{1-2v^2}$ B. $\dfrac{2}{1-v^2}$ C. $\dfrac{2v^2}{(1-v^2)^2}$ D. $\dfrac{2}{(1-v)^2}$ E. $\dfrac{1}{d^{(1/2)}}$

Short answer:
If you happen to know that the annual perpetuity has a duration of $1/d$, then it follows that the duration for the biannual duration should be $2\left(1/d^{(1/2)}\right)$. This is because $1/d^{(1/2)}$ should be the duration of the biannual perpetuity counted in two-year periods, and to obtain a result in years, as duration must always be in units of years, we must multiply by 2. Now if we notice that $1-v^2 = d^{(1/2)}$, we can see that the correct choice is B.

Longer answer 1:
We can compute from scratch.

$$\begin{aligned}\text{MacD} = \frac{\sum tA_t}{\sum A_t} &= \frac{2v^2+4v^4+6v^6+\cdots}{v^2+v^4+v^6+\cdots}\\ &= \frac{2(Ia)_{\overline{\infty}|j}}{a_{\overline{\infty}|j}}\end{aligned}$$

where j is the effective rate of interest per 2 years. Let d_j be the effective rate of discount per 2 years

$$\begin{aligned}&= \frac{2\frac{1}{d_j\times j}}{\frac{1}{j}}\\ &= \frac{2}{d_j}\\ &= \boxed{\frac{2}{1-v^2}}\end{aligned}$$

Long answer 2:
We can compute from scratch, and practice with power series from calculus is useful in this case.

$$\begin{aligned}\text{MacD} = \frac{\sum tA_t}{\sum A_t} &= \frac{2v^2+4v^4+6v^6+\cdots}{v^2+v^4+v^6+\cdots}\\ &= \frac{v\left(2v+4v^3+\cdots\right)}{\left(\sum_{k=0}^{\infty} v^{2k}\right)-1}\\ &= \frac{v\left(\frac{d}{dv}\left(v^2+v^4+\cdots\right)\right)}{\frac{1}{1-v^2}-1}\\ &= \frac{v\left(\frac{d}{dv}\left(\left(\sum_{k=0}^{\infty} v^{2k}\right)-1\right)\right)}{(1-v^2)^{-1}-1}\end{aligned}$$

$$= \frac{v\left(\frac{d}{dv}\left(\frac{1}{1-v^2}-1\right)\right)}{(1-v^2)^{-1}-1}$$

$$= \frac{v\left(-1(1-v^2)^{-2}(-2v)\right)}{(1-v^2)^{-1}-1}$$

$$= \frac{2v^2\left(1-v^2\right)^{-2}}{(1-v^2)^{-1}-1}$$

$$= \frac{2v^2\left(1-v^2\right)^{-1}}{1-(1-v^2)}$$

$$= \boxed{\frac{2}{1-v^2}}$$
