The Infinite Actuary Exam S Online Seminar
CAS S C.4.2 Problems with Solutions

1. First Pass:
The probability distribution function of claims per year for an individual risk is a Poisson distribution with parameter λ. The prior distribution of λ is a gamma distribution with density

$$f(\lambda) = \lambda e^{-\lambda}, \qquad 0 < \lambda < \infty.$$

Given an observation of one claim in a one-year period, what is the density of the posterior distribution of λ?

A. $e^{-\lambda}$ B. $\dfrac{\lambda^2}{2}e^{-\lambda}$ C. $4\lambda^2 e^{-2\lambda}$ D. $\lambda e^{-2\lambda}$ E. $4\lambda e^{-2\lambda}$

Our prior is a Gamma with $\theta = 1$ and $\alpha = 2$. The posterior has $\alpha' = \alpha + k = 2 + 1 = 3$, and $1/\theta' = 1/\theta + n = 2$, so $\theta' = 1/2$ and thus our posterior density is $\boxed{4\lambda^2 e^{-2\lambda}}$

This is #46 from the 1990 Spring exam 4.

---

2. For a large number of health insurance policies the annual numbers of claims follow the Poisson distribution with mean λ, which varies by policy. The prior probability density function of λ is

$$h(\lambda) = \left(\frac{1}{2}\right)\lambda^2 e^{-\lambda}, \lambda > 0$$

In Year 1, four claims are reported for a policy.

Calculate the expected value of the posterior distribution of λ for this policy.

A. Less than 2.0
B. At least 2.0, but less than 3.0
C. At least 3.0, but less than 4.0
D. At least 4.0, but less than 5.0
E. At least 5.0

We observe that the prior distribution of λ is a Gamma distribution with $\theta = 1$ and $\alpha = 3$.. Since we observe 4 claims in 1 year, the posterior distribution for λ is Gamma with $\alpha' = \alpha + 4 = 7$, and $\frac{1}{\theta'} = \frac{1}{\theta} + 1 \longrightarrow \theta' = 1/2$. The expected value of this new Gamma distribution is $7 \cdot 0.5 = 3.5$, answer choice C.

3. You are given:

- An individual automobile insured has an annual claim frequency distribution that follows a Poisson distribution with mean λ.
- λ follows a gamma distribution with parameters α and θ.
- The first actuary assumes that $\alpha = 1$ and $\theta = 1/6$.
- The second actuary assumes the same mean for the gamma distribution, but only half the variance.
- A total of one claim is observed for the insured over a three year period.
- Both actuaries determine the posterior mean for λ using their model assumptions.

Determine the ratio of the posterior mean that the first actuary calculates to the posterior mean that the second actuary calculates.

A. 3/4 B. 9/11 C. 10/9 D. 11/9 E. 4/3

Actuary 1 has a posterior distribution that is a Gamma with $\alpha' = \alpha + 1$ and $1/\theta' = 1/\theta + 3$, so $\alpha' = 2$ and $\theta' = 1/9$.

Actuary 2 has $\alpha\theta = 1/6$ and $\alpha\theta^2 = 1/72$, so $\alpha = 2$ and $\theta = 1/12$. The posterior for actuary 2 is thus a Gamma with $\alpha' = 2 + 1 = 3$ and $1/\theta' = 12 + 3 = 15$, so $\theta' = 1/15$. The ratio of the posterior means is thus
$\dfrac{2 \cdot \frac{1}{9}}{3 \cdot \frac{1}{15}} = \boxed{\dfrac{10}{9}}$.

4. The probability distribution function of claims per year for an individual risk is a Poisson distribution with parameter λ. The prior distribution of λ is a gamma distribution with density

$$f(\lambda) = \lambda e^{-\lambda}, \qquad 0 < \lambda < \infty.$$

Given an observation of one claim in a one-year period, what is the posterior probabilty that λ exceeds 2?

A. Less than 0.1
B. At least 0.1, but less than 0.2
C. At least 0.2, but less than 0.3
D. At least 0.3, but less than 0.4
E. At least 0.4

Our prior is a Gamma with $\theta = 1$ and $\alpha = 2$. The posterior has $\alpha' = \alpha + k = 2 + 1 = 3$, and $1/\theta' = 1/\theta + n = 2$. The probability that a Gamma $(3, 0.5) > 2$ is the same as the probability that a Poisson process with rate $1/0.5 = 2$ has less than 3 arrivals by time 2. This is because the Gamma random variable has the same distribution as a sum of 3 independent exponential random variables with mean 1/2. The distribution of the number of Poisson events occuring before time 2 is Poisson with rate $2 \cdot 2 = 4$. The probability that this is less than 3 is $e^{-4}(1 + 4 + 4^2/2) \approx \boxed{0.238}$.

---

5. You are given the following information for an insurance policy:

- Monthly claim frequencies follow a Poisson process with parameter λ.
- The prior distribution of λ follows a gamma distribution with parameters $\alpha = 3$ and $\theta = 2$.
- In the first month, a policy had 27 claims.

Calculate the posterior mean monthly claims for this policy.

A. Less than 9
B. At least 9, but less than 13
C. At least 13, but less than 17
D. At least 17, but less than 21
E. At least 21

This is a Poisson/Gamma pair with $k = 27$ claims observed in $m = 1$ period. The posterior is Gamma with $\alpha' = 3 + 27 = 30$, and $\theta = \frac{1}{1/2+1} = 2/3$. The mean of this posterior is $30 \cdot \frac{2}{3} = \boxed{20}$, which leads to answer choice D.

This is #22 from the Spring 2014 ST exam.

6. You are given the following information:

- $X_1, \ldots, X_5$ are a random sample from a Poisson distribution with parameter λ, where λ follows a gamma distribution with parameters $\alpha = 2$ and θ.
- The mean of this Poisson-gamma conjugate pair can be represented as a weighted average of the maximum likelihood estimator for the mean and the mean of the prior distribution.
- Let W_{MLE} be the weight assigned to the maximum likelihood estimator.
- The maximum likelihood estimate for the mean is 1.2.
- The variance of the prior gamma is 8.

Calculate W_{MLE}.

A. Less than 0.60
B. At least 0.60, but less than 0.70
C. At least 0.70, but less than 0.80
D. At least 0.80, but less than 0.90
E. At least 0.90

From bullet one, the prior distribution is Gamma with $\alpha = 2$. From bullet five, the variance of this gamma is 8, which allows us to compute θ.

$$\begin{aligned} \text{Var(Prior)} &= \alpha\theta^2 \\ 8 &= 2 \cdot \theta^2 \\ \theta &= 2. \end{aligned}$$

The average of the prior is therefore $2 \cdot 2 = 4$.

I'm assuming that the mean of the conjugate pair refers to the mean of the posterior or predicitive distribution, which are the same for the Poisson/Gamma case. Since the MLE for the mean is $\bar{X}$, we find that the number of exposures is $n = 5$ and the number of claims observed is $n\bar{X} = 5 \cdot 1.2 = 6$. The posterior is therefore Gamma with $\alpha' = 2 + 6$ and $\theta' = \frac{1}{1/2+5} = 2/11$. The mean of the posterior is $8 \cdot 2/11 = 16/11$.

To find the weight W_{MLE}, set up the weighted average, set it equal to 16/11, and solve.

$$\begin{aligned} 16/11 &= W_{\text{MLE}}(1.2) + (1 - W_{\text{MLE}})4 \\ (4 - 1.2)W_{\text{MLE}} &= 4 - 16/11 \\ W_{\text{MLE}} &= \frac{4 - 16/11}{4 - 1.2} = 10/11 = 0.\overline{90} \end{aligned}$$

This result lies in the range of answer choice E.

This is #23 from the Spring 2014 ST exam.

---

7. You are given:

- The number of claims per year for a policyholder follows a Poisson distribution with mean λ.
- The prior distribution for the mean, λ, follows a gamma distribution with mean 0.2 and variance 0.005.
- Last year, 60 annual policies produced a total of 12 claims.

Calculate the variance of the posterior distribution of λ.

A. Less than 0.0015
B. At least 0.0015, but less than 0.0025
C. At least 0.0025, but less than 0.0035
D. At least 0.0035, but less than 0.0045
E. At least 0.0045

Start by finding α and θ

$$\begin{aligned}
\mathrm{E}[\lambda] &= 0.2 \\
\alpha\theta &= 0.2 \\
\mathrm{Var}[\lambda] &= \alpha\theta^2 = (0.2)\theta \\
0.005 &= 0.2\theta \longrightarrow \theta = 0.025,\ \alpha = 8.
\end{aligned}$$

We made 60 observations of the Poisson, and saw 12 total claims, so

$$\alpha' = 8 + 12 = 20, \quad \frac{1}{\theta'} = \frac{1}{0.025} + 60 = 100.$$

The posterior $\theta' = 1/100$. The variance of the posterior is $\alpha'\theta'^2 = 20(1/100)^2 = \boxed{0.002}$.

This is problem #23 from the Spring 2015 ST exam.

---

8. You are given:

- Daily claim counts follow a Poisson distribution with mean λ.
- The prior distribution of λ has probability density function:

$$f(\lambda) = \frac{1}{5}e^{-\lambda/5}$$

- Three claims were observed on the first day.

Calculate the variance of the posterior distribution of λ.

A. Less than 1.0
B. At least 1.0, but less than 2.0
C. At least 2.0, but less than 3.0
D. At least 3.0, but less than 4.0
E. At least 4.0

The given prior is an exponential distribution with mean 5, which is also a Gamma distribution with $\alpha = 1$ and $\theta = 5$. We observe 3 on a single instance of the distribution, so the posterior distribution of λ is a Gamma distribution with $\alpha' = 1 + 3 = 4$, and $\frac{1}{\theta'} = \frac{1}{\theta} + 1 \longrightarrow \theta' = \frac{5}{6}$. The variance of this posterior Gamma distribution is $\alpha'\theta'^2 = 4\left(\frac{5}{6}\right)^2 = \boxed{2.\overline{7}}$.
This is #25 from the Fall 2015 ST exam.

---

9. The number of claims for an individual risk in a single year follows a Poisson distribution with parameter λ. The parameter λ has for a prior distribution the following gamma density function with parameters $\alpha = 1$ and $\theta = 2$:

$$f(\lambda) = \frac{1}{2}e^{-\frac{\lambda}{2}},\ \lambda > 0.$$

You are given that three claims arose in the first year.

Determine the posterior distribution of λ.

A. $\frac{1}{2}e^{-\frac{3}{2}\lambda}$ B. $\frac{1}{12}\lambda^3 e^{-\frac{1}{2}\lambda}$ C. $\frac{1}{4}\lambda^3 e^{-\frac{1}{2}\lambda}$ D. $\frac{27}{32}\lambda^3 e^{-\frac{3}{2}\lambda}$ E. $\frac{1}{12}\lambda^2 e^{-\frac{3}{2}\lambda}$

We have a Poisson/Gamma conjugate prior with $\alpha = 1, \theta = 2$. We observe $k = 3$ total claims in $m = 1$ observation, so the posterior distribution of λ is Gamma with $\alpha' = 1 + 3 = 4$, $\frac{1}{\theta'} = \frac{1}{2} + 1 \longrightarrow \theta' = \frac{2}{3}$. The density of the posterior is $C\lambda^{4-1}e^{-\lambda/(2/3)}$ which, without even worrying about the constant, narrows our answer choice down to only D.

This is #28 from the Spring 1992 CAS Exam 4B.

---

10. The number of claims for an individual risk in a single year follows a Poisson distribution with parameter λ. The parameter λ has for a prior distribution the following gamma density function with parameters $\alpha = 1$ and $\theta = 2$:

$$f(\lambda) = \frac{1}{2}e^{-\frac{\lambda}{2}},\ \lambda > 0.$$

    You are given that three claims arose in the first year.

    Determine the Bayesian estimate for the expected number of claims in the second year.

    A. Less than 2.25
    B. At least 2.25, but less than 2.50
    C. At least 2.50, but less than 2.75
    D. At least 2.75, but less than 3.00
    E. At least 3.00

We have a Poisson/Gamma conjugate prior with $\alpha = 1, \theta = 2$. We observe $k = 3$ total claims in $m = 1$ observation, so the posterior distribution of λ is Gamma with $\alpha' = 1 + 3 = 4$, $\frac{1}{\theta'} = \frac{1}{2} + 1 \longrightarrow \theta' = \frac{2}{3}$. The density of the posterior is $C\lambda^{4-1}e^{-\lambda/(2/3)}$.

For the Poisson/Gamma, the Baysian estimate for the expected number of claims is the same as the mean of the posterior, $4 \cdot \frac{2}{3} = \boxed{2.\overline{6}}$.

This is #29 from the Spring 1992 CAS Exam 4B.

---

11. You are given the following:

- For an Individual risk In a population, the number of claims for a single exposure period follows a Poisson distribution with mean λ.
- For the population, λ is distributed according to an exponential distribution with mean 0.1;

$$g(\lambda) = 10e^{-10\lambda}, \lambda > 0$$

- An individual risk is selected at random from the population.
- After one exposure period, one claim has been observed.

Determine the density function of the posterior distribution of λ for the selected risk.

A. $11e^{-11\lambda}$ B. $10\lambda e^{-11\lambda}$ C. $121\lambda e^{-11\lambda}$ D. $\frac{1}{10}e^{-9\lambda}$ E. $\frac{11e^{-11\lambda}}{\lambda^2}$

An exponential distribution with mean 0.1 is gamma with $\alpha = 1, \theta = 1/10$. Thus, we have a Poisson/Gamma prior, and with 1 observed claim for 1 exposure, the posterior will also be gamma, but with $\alpha' = 1 + 1 = 2$, and $\frac{1}{\theta'} = \frac{1}{1/10} + 1 \longrightarrow \theta' = 1/11$.

The density function for this new gamma is

$$C\lambda^{\alpha'-1}e^{-\lambda/\theta'} = C\lambda^1 e^{-11\lambda},$$

but this only reduces the answer choices to B and C. To find the constant, we can either refer to the exam tables to find the formula

$$\frac{(\lambda/\theta')^{\alpha'}e^{-\lambda/\theta}}{\lambda\Gamma(\alpha')} = \frac{(11\lambda)^2 e^{-11\lambda}}{\lambda(2-1)!} = 121\lambda^1 e^{-11\lambda},$$

or we can integrate and set the resulting total probability to 1. In the later method, we avoid integration by parts by creating an integral that represents the expected value of an exponential random variable.

$$1 = \int_0^\infty C\lambda e^{-11\lambda}\,d\lambda = C \cdot \frac{1}{11}\int_0^\infty \lambda \cdot 11 \cdot e^{-11\lambda}\,d\lambda = C \cdot \frac{1}{11} \cdot \frac{1}{11} \longrightarrow C = 121.$$

This is #25 from the Spring 1994 CAS exam 4B.

---

12. You are given the following:

- The number of claims per year for a given risk follows a Poisson distribution with mean λ.
- The prior distribution of λ is assumed to be a gamma distribution with mean 1/2 and variance 1/8.

Determine the variance of the posterior distribution of λ if a total of 4 claims have been observed for this risk in a 2-year period.

A. $\dfrac{1}{16}$ B. $\dfrac{1}{8}$ C. $\dfrac{1}{6}$ D. $\dfrac{1}{2}$ E. 1

We need to find the parameters for the original gamma distribution.

$$\alpha\theta = \frac{1}{2}, \qquad \alpha\theta^2 = \frac{1}{2}\cdot\theta = \frac{1}{8} \longrightarrow \theta = \frac{1}{4},\ \alpha = 2.$$

Since this is a Poisson/Gamma conjugate prior pair, having observed 4 claims in 2 observations, the posterior is a gamma distribution with $\alpha' = 2 + 4 = 6$, and $\frac{1}{\theta'} = \frac{1}{1/4} + 2 \longrightarrow \theta' = \frac{1}{6}$.

The variance of this posterior distribution is $\alpha'\theta'^2 = 6 \cdot \left(\dfrac{1}{6}\right)^2 = \boxed{\dfrac{1}{6}}$.

This is # 21 from the Spring 1996 4B exam.

13. You are given the following:

r is a random variable that represents the number of claims for an individual risk and has the Poisson density function

$$f(r) = \frac{t^r e^{-t}}{r!}, r = 0, 1, 2, ...$$

The parameter t has a prior gamma distribution with density function

$$h(t) = 5e^{-5t}, t > 0$$

A portfolio consists of 100 independent risks, each having identical density functions.

In one year 10 claims are experienced by the portfolio.

Determine the Bayesian expected number of claims in the second year for the portfolio.

A. Less than 6
B. At least 6, but less than 8
C. At least 8, but less than 10
D. At least 10, but less than 12
E. At least 12

We are left to assum that the Poisson is for the number of claims in a given year, but this is not unreasonable. The given gamma distribution has $\alpha = 1, \theta = 1/5$. With 10 claims in 100 observations the posterior of this Poisson/Gamma conjugate prior pair is also gamma with $\alpha' = 1 + 10 = 11$, and $\frac{1}{\theta'} = \frac{1}{1/5} + 100 \longrightarrow \theta' = \frac{1}{105}$.

The expected number of claims for the portfolio in the second year will be 100 times the mean of the predictive distribution, which for the Poisson/Gamma this is the same as the mean of the posterior,

$$100 \cdot \alpha'\theta' = 100 \cdot 11 \cdot \frac{1}{105} \approx \boxed{10.476}$$

This is #24 from the fall 1994 CAS 4B exam.

14. An individual automobile insured has a claim count distribution per policy period that follows a Poisson distribution with parameter λ. For the overall population of insureds, λ follows a distribution with density function

$$f(\lambda) = 5exp(-5\lambda), \lambda > 0$$

One insured is selected at random from the population and is observed to have a total of one claim during two policy periods.

Determine the expected number of claims that this same insured will have during the third policy period.

A. Less than 0.2
B. At least 0.2, but less than 0.4
C. At least 0.4, but less than 0.6
D. At least 0.6, but less than 0.8
E. At least 0.8

The given gamma distribution has $\alpha = 1, \theta = 1/5$. With 1 claim observed in two exposures for a single insured, the posterior distribution is gamma with $\alpha' = 1 + 1 = 2$, and $\frac{1}{\theta'} = \frac{1}{1/5} + 2 \longrightarrow \theta' = \frac{1}{7}$.

We are looking for the posterior mean claims for a single individual, which for the Poisson/Gamma pair is the same as the mean of the posterior, $\alpha' \cdot \theta' = \boxed{2/7 \approx 0.286}$.

This is #4 from a sample exam that was released for Course 4 in 2000, with made up answer choices.

15. You are given the following:

- A portfolio consists of 1000 identical and independent risks.
- The number of claims for each risk follows a Poisson distribution with mean λ.
- Prior to the latest exposure period, λ is assumed to have a gamma distritbution, with parameiers $\alpha = 250$ and $\theta = 0.0005$.
- During the latest exposure period, the following loss experience is observed:

Number of Claims	Number of Risks
0	906
1	89
2	4
3	1
	1000

Determine the mean of the posterior distribution of λ.

A. Less than 0.11
B. At least 0.11, but less than 0.12
C. At least 0.12, but less than 0.13
D. At least 0.13, but less than 0.14
E. At least 0.14

The total number of observed claims is $89 + 2(4) + 3(1) = 100$, and the total number of exposures is given as 1000, so the posterior distribution for this Poisson/Gamma pair is gamma with $\alpha' = 250 + 100$ and $\frac{1}{\theta'} = \frac{1}{0.0005} + 1000 \longrightarrow \theta' = 1/3000$.

The mean of this posterior is $\alpha'\theta' = 350/3000 \approx \boxed{0.1167}$.

This is #12 from the Spring 1995 CAS 4B exam.

16. You are given the following:

- A portfolio consists of 100 identical and independent risks.
- The number of claims per year for each risk follows a Poisson distribution with mean λ.
- The prior distribution of λ is assumed to be a gamma distribution with mean 0.25 and variance 0.0025.
- During the latest year, the following loss experience is observed:

Number of Claims	Number of Risks
0	80
1	17
2	3

Determine the variance of the posterior distribution of λ.

A. Less than 0.00075
B. At least 0.00075, but less than 0.00125
C. At least 0.00125, but less than 0.00175
D. At least 0.00175, but less than 0.00225
E. At least 0.00225

First find the parameters of the prior gamma distribution.

$$\alpha\theta = 0.25 \qquad \alpha\theta^2 = 0.25\theta = 0.0025 \longrightarrow \theta = 1/100,\ \alpha = 25$$

A total of $17 + 2(3) = 23$ claims were observed in 100 exposures, so the posterior of this Poisson/Gamma pair is gamma with $\alpha' = 25 + 23 = 48$ and $\frac{1}{\theta'} = \frac{1}{1/100} + 100 \longrightarrow \theta' = 1/200$.

The variance of this posterior distribution is $\alpha'\theta'^2 = 48/(200^2) \approx \boxed{0.0012}$

This is #2 from the Fall 1997 CAS exam 4B.

---

17. You are given:

- Annual claim counts follow a Poisson distribution with mean λ.
- The parameter λ has a prior distribution with probability density function:

$$f(\lambda) = \frac{1}{3}e^{-\lambda/3}, \lambda > 0$$

Two claims were observed during the first year.

Determine the variance of the posterior distribution of λ.

A. $\dfrac{9}{16}$ B. $\dfrac{27}{16}$ C. $\dfrac{9}{4}$ D. $\dfrac{16}{3}$ E. $\dfrac{27}{4}$

The prior distribution is gamma with $\alpha = 1$ and $\theta = 3$. Since 2 claims were observed in one exposure, the posterior of this Poisson/Gamma pair is gamma with $\alpha' = 1 + 2$, and $\frac{1}{\theta'} = \frac{1}{3} + 1 \longrightarrow \theta' = 3/4$.

The variance of this posterior is $\alpha'\theta'^2 = 3 \cdot \left(\frac{3}{4}\right)^2 = \boxed{\frac{27}{16}}$.

This is #2 from the Spring 2001 Course 4 exam.

18. You are given:

- The number of claims per auto insured folows a Poisson disbibution with mean λ.
- The prior distribution for λ has the following probability density function:

$$f(\lambda) = \frac{(500\lambda)^{50} e^{-500\lambda}}{\lambda \Gamma(50)}$$

- A company observes the following claims experience:

	Year 1	Year 2
Number of claims	75	210
Number of autos insured	600	900

The company expects to insure 1100 autos in Year 3.

Determine the expected number of claims in Year 3.

A. 178 B. 184 C. 193 D. 209 E. 224

The provided prior distribution is gamma with $\alpha = 50,\ \theta = 1/500$. With $210 + 75 = 285$ claims observed in $600 + 900 = 1500$ exposures, the posterior distribution of this Poisson/Gamma pair is gamma with $\alpha' = 50 + 285$ and $\frac{1}{\theta'} = \frac{1}{1/500} + 1500 \longrightarrow \theta' = 1/2000$.

The expected number of claims per policy in the following year, since the mean of the predictive distribution is the same as the posterior for the Poisson/Gamma, is $\alpha'\theta' = 335/2000$. Since the company expects to insure 1100 autos in year 3, the total expected claims will be $1100 \cdot \dfrac{335}{2000} \approx \boxed{184.25}$

This is #3 from the Fall 2001 Course 4 exam.

---

19. The annual number of claims for a policy follows the Poisson distribution with mean λ, which varies by policy. The prior probability density function of λ is

$$h(\lambda) = \lambda e^{-\lambda}, \quad \lambda > 0$$

In Year 1, three claims are reported for a policy.

Calculate the coefficient of variation (the standard deviation divided by the mean) of the posterior distribution of λ, for this policy.

A. Less than 0.5
B. At least 0.5, but less than 1.0
C. At least 1.0, but less than 1.5
D. At least 1.5, but less than 2.0
E. At least 2.0

By comparing the given density with the density for the gamma distribution in the tables, determine that $\alpha = 2$, and $\theta = 1$. We observe one policy for one year and witness 3 claims. Therefore, the posterior is a gamma distribution with

$$\alpha' = 2 + 3 = 5, \quad \frac{1}{\theta'} = \frac{1}{1} + 1 = 2 \longrightarrow \theta' = \frac{1}{2}.$$

Finally, the coefficient of variation is

$$\frac{\sqrt{\mathrm{Var}(\lambda)}}{E[\lambda]} = \frac{\sqrt{\alpha'\theta'^2}}{\alpha'\theta'} = \frac{\sqrt{5/4}}{5/2} \approx \boxed{0.45}.$$

This is #24 from the Spring 2016 exam ST.

---

20. You are given:

- The number of claims per year for a policyholder follows a Poisson distribution with mean λ.
- The mean λ follows a gamma distribution with parameters $\alpha = 3$ and $\theta = 10$.
- A policyholder had 1 claim last year.

Calculate the mean of the posterior distribution of λ.

A. Less than 2.0
B. At least 2.0, but less than 4.0
C. At least 4.0, but less than 6.0
D. At least 6.0, but less than 8.0
E. At least 8.0

Since we observed one claim on one observations, the posterior is a gamma distribution with

$$\alpha' = 3 + 1 = 4, \quad \frac{1}{\theta'} = \frac{1}{10} + 1 \longrightarrow \theta' = \frac{10}{11}.$$

The mean of this distribution is $4 \cdot 10/11 \approx \boxed{3.6}$.

This is #25 from the Spring 2016 exam ST.
