Review - Outline

1. A.1.1 Review
   - Continuously Compounded Returns
   - Prepaid Forward Contracts
   - Call and Put Options

Interest Rate Compounding

If r is quoted as an **effective** annual interest rate, then if you invest $\$X$ today, in t years you will have $X(1+r)^t$

If r is quoted as a **continuously compounded** annual interest rate, then if you invest $\$X$ today, in t years you will have Xe^{rt}

> If you purchase asset S at time t for price S_t and sell it for price S_{t+h} in the future, then your continuously compounded return on the investment for period h must solve:
>
> $$\begin{aligned} S_t e^r &= S_{t+h} \\ r &= \ln(S_{t+h}/S_t) \end{aligned}$$

Return Volatility

An asset's **volatility**, σ, generally refers to the sample standard deviation of its returns. Given returns $r_1, r_2, r_3, ..., r_N$, volatility can be calulated as:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(r_i - \bar{r})^2}$$

Note that:

- The formula inside the radical gives the sample variance, σ^2
- $\bar{r}$ is the sample average. I.e., $\bar{r} = \dfrac{1}{N}\sum_{i=1}^{N} r_i$

Properties of Continuously Compounded Returns

Let $r_h, r_{2h}, r_{3h}, ..., r_{nh}$ be the continuously compounded returns measured at frequency h on asset S between times 0 and T, where $h = T/n$ is measured in years. Then:

1. Increases and decreases are symmetric
   - If $r_h = R$ and $r_{2h} = -R$, then $S_{2h} = S_0 e^R e^{-R} = S_0$
2. Returns are additive
   - $\ln\left(\frac{S_T}{S_0}\right) = \sum_{i=1}^{n} r_{ih}$
3. Volatility is proportional to the square root of time
   - E.g., let σ_h be the volatility of the returns measured at frequency h. Then $\sigma = \frac{\sigma_h}{\sqrt{h}}$, where σ is the annual volatility.
   - This implies variance is proportional to time

Forward vs. Prepaid Forward Contract

Forward contract – contract to buy or sell an asset at a specific price on a specific future delivery date

- Price is agreed upon today, but settlement occurs on the delivery date
- Denote forward price today to purchase asset S on date T as $F_{0,T}(S)$
- For the purposes of this exam, a forward contract and a futures contract are synonymous

Prepaid forward contract – forward contract that is settled today

- Denote price today of a prepaid forward to purchase asset S on date T as $F^P_{0,T}(S)$

Prepaid Forward Formulas

- Cash

$$F^P_{0,T}(\$1) = \$1e^{-rT}$$

- Foreign Currency

$$\$F^P_{0,T}(1¥) = x_0 e^{-r_{¥}T}$$

where x_0 is the \$/¥ exchange rate and $r_{¥}$ is the risk-free rate for yen

- Coupon Bond

$$F^P_{0,T}(B) = B_0 - \sum_{t=0}^{T} \text{coupon}_t \, e^{-rt}$$

where B_0 is the bond price today

Prepaid Forward Formulas *(continued)*

- Stock
  - No dividends

$$F^P_{0,T}(S) = S_0$$

  - Discrete dividends

$$F^P_{0,T}(S) = S_0 - \sum_{t=0}^{T} \text{dividend}_t \, e^{-rt}$$

  - Continuous dividends

$$F^P_{0,T}(S) = S_0 \, e^{-\delta T}$$

where δ is the stock's continuous dividend yield

- Forward/Futures Contract

$$F^P_{0,T}(F) = F_{0,T} \, e^{-rT}$$

Prepaid Forward Formulas: Examples

Example 1

A stock has a price of \$50 today and pays a dividend of \$2 every 6 months. The next dividend will occur 3 months from today. Assume the continuously compounded annual risk-free rate is 5%. Find the price today of a prepaid forward contract for delivery of 1 share of the stock in 1 year.

- Since the dividends in 3 months and 9 months will occur between now and the delivery date, we have:

$$F_{0,1}^{P}(S) = 50 - 2e^{-.05(3/12)} - 2e^{-.05(9/12)} = \mathbf{46.10}$$

Example 2

For the same stock, find the price today of a prepaid forward contract for delivery of 1 share of the stock in 2 months.

- In this case, there are no dividends paid between now and the delivery date, so:

$$F^P_{0,2/12}(S) = \mathbf{50}$$

Example 3

Let the price of the same stock in eight months be $S_{8/12}$. What price, expressed as a function of $S_{8/12}$, would you pay in 8 months for delivery of the stock one year from today?

- We are being asked for $F^P_{8/12,1}(S)$
- The dividend occurring 9 months from today will be the only dividend paid between the prepayment date (8 months) and the delivery date (1 year). In eight months, that dividend will only be one month away. Thus:

$$F^P_{8/12,1}(S) = S_{8/12} - 2e^{-.05(1/12)}$$

A **call option** with strike $\$K$ on underlying asset S gives the holder (i.e., the long position) the right, but not the obligation, to give up $\$K$ in exchange for asset S

$$\text{payoff} = \max(0, S - K)$$

A **put option** with strike $\$K$ on underlying asset S gives the holder the right, but not the obligation, to give up S in exchange for $\$K$

$$\text{payoff} = \max(0, K - S)$$

Option Basics

Options have expiration dates (T)

- **European options** can be exercised only at time T
- **American options** can be exercised at any time $t \leq T$

We denote an option's price (i.e., the option's **premium**) as:

- $C(S, K, T)$ for calls
- $P(S, K, T)$ for puts

Let S_t be the underlying stock price at time t. At time t, an option is said to be:

- **at the money** if $S_t = K$
- **in the money** if $S_t > K$ for a call or $S_t < K$ for a put
- **out of the money** if $S_t < K$ for a call or $S_t > K$ for a put

The following table gives the month-end prices of a non-dividend paying stock for five consecutive months:

Month	Price
Jan	34
Feb	31
Mar	34
Apr	36
May	37

Estimate the annual volatility for this stock.

A. 7% B. 8% C. 16% D. 24% E. 28%

1. Use calculator to calculate the cont. compounded returns
2. Use the stat menu to find $\sigma_{monthly} = .0802$
3. Annualize the volatility

$$\sigma_{annual} = \sigma_{monthly} \times \sqrt{12} = 0.2778$$