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An Asian option's payoff depends on the average price of the underlying stock over the life of the option. The average can take the place of underlying asset price (“average price” option) or the strike price (“average strike” option) in determining the payoff.

An Asian option can vary in whether it is:

- ① a call or a put
- ② an average price or average strike option
- ③ based on arithmetic or geometric averaging

Thus, there are 8 possible Asian option variations to consider



Assume the option is created at time 0. Let h be the option's averaging frequency and T be the option's time to expiration. The average will have $N = T/h$ observations, with the last observation occurring at time T .

There are two ways to compute the average price:

- ① Arithmetic average:
$$A(T) = \frac{1}{N} \sum_{i=1}^N S_{ih}$$
- ② Geometric average:
$$G(T) = \left[\prod_{i=1}^N (S_{ih}) \right]^{1/N}$$

It turns out that $G(T) \leq A(T)$ for all possible price paths



Option	Payoff
Arithmetic average price call	$\max(0, A(T) - K)$
Arithmetic average strike call	$\max(0, S_T - A(T))$
Arithmetic average price put	$\max(0, K - A(T))$
Arithmetic average strike put	$\max(0, A(T) - S_T)$
Geometric average price call	$\max(0, G(T) - K)$
Geometric average strike call	$\max(0, S_T - G(T))$
Geometric average price put	$\max(0, K - G(T))$
Geometric average strike put	$\max(0, G(T) - S_T)$



The price of an Asian option is \leq an otherwise identical ordinary European option

The volatility of $\ln(S/K)$ is smaller when the average stock price is used in place of S or K

For average price options:

- If $N = 1$, Asian option = European option
 - The only observation in the average would be S_T
- Increasing N decreases the Asian option's price

For average strike options:

- If $N = 1$, the Asian option's value is 0
- Increasing N increases the Asian option's price



Example 1

On January 1, a stock is priced at \$45, and an at-the-money arithmetic average price Asian call option expiring on April 30 of the same year is created. Assume the stock price at the end of the next four months is \$50, \$60, \$70 and \$60. Find the payoff at expiration of the Asian option assuming it uses monthly averaging.

- $A(T) = \frac{50+60+70+60}{4} = 60$
- Payoff = $\max(A(T) - K, 0) = \max(60 - 45, 0) = \mathbf{15}$



Example 2

On January 1, a geometric average strike Asian put option expiring on April 30 of the same year is created. The underlying stock is the stock in the previous example. Find the payoff at expiration of the Asian option assuming it uses monthly averaging.

- $G(T) = (50 \times 60 \times 70 \times 60)^{1/4} = 59.5789$
- $\text{Payoff} = \max(G(T) - S_T, 0) = \max(59.5789 - 60, 0) = 0$