A.2.3 Life Tables

Notation

Example

Exam Tables

Exercises

Life Tables Notation

- l_x - expected number of survivors at (x)
  - $\ell_x \equiv l_x$
- ${}_nd_x$ - expected number of deaths between exact ages x and $x+n$
- d_x - expected number of deaths between exact ages x and $x+1$

Life Table Example

x	l_x	
0	81	radix
1	64	
2	49	
3	36	
4	25	
5	16	
6	9	
7	4	
8	1	
9	0	ω

$$l_x = l_0 \cdot S_0(x)$$

$$S_0(4) = \frac{25}{81}$$

$$p_x = \frac{l_{x+1}}{l_x} \qquad q_x = \frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x}$$

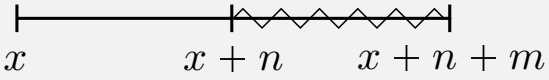
$$_np_x = \frac{l_{x+n}}{l_x} \qquad _nq_x = \frac{_nd_x}{l_x} = \frac{l_x - l_{x+n}}{l_x}$$

$$_2p_5 = \frac{4}{16} \qquad _3q_1 = \frac{64 - 25}{64} = \frac{39}{64}$$

Life Table Example cont.

x	l_x	
0	81	radix
1	64	
2	49	
3	36	
4	25	
5	16	
6	9	
7	4	
8	1	
9	0	ω

$$_{n|m}q_x = \frac{l_{x+n} - l_{x+n+m}}{l_x}$$



$$_{2|}q_3 = \frac{l_5 - l_6}{l_3} = \frac{16 - 9}{36} = \frac{7}{36}$$

Table built from

$$S_0(x) = \left(\tfrac{9-x}{9}\right)^2$$

$$l_x = 81\left(\tfrac{9-x}{9}\right)^2$$

$_{2.5}p_3$ cannot be determined from table

Exam Tables

LTAM – Standard Ultimate Life Table

MLC – Illustrative Life Table

Exercise 1

You are given the following mortality table:

Age(x)	q_x	l_x	d_x
20		30,000	1,200
21			
22		27,350	
23	0.0700		
24	0.0790	23,900	

Determine the probability that a life aged 21 will die within two years.

Exercise 1

You are given the following mortality table:

Age(x)	q_x	l_x	d_x
20		30,000	1,200
21			
22		27,350	
23	0.0700		
24	0.0790	23,900	

Determine the probability that a life aged 21 will die within two years.

$$
\begin{aligned}
{}_2q_{21} &= \frac{l_{21}-l_{23}}{l_{21}} = \frac{28800-25699}{28800} = \boxed{0.1077} \\
l_{21} &= l_{20} - d_{20} = 30000 - 1200 = 28800 \\
l_{23} \times p_{23} &= l_{24} \\
l_{23} &= \frac{23900}{0.93} = 25699
\end{aligned}
$$

Exercise 2

(same exercise from previous lesson, but solved using life table now)

You are given ${}_{1|}q_{x+1} = 0.095$, ${}_{2|}q_{x+1} = 0.171$ and $q_{x+3} = 0.200$. Calculate $q_{x+1} + q_{x+2}$.

Exercise 2

You are given ${}_{1|}q_{x+1} = 0.095$, ${}_{2|}q_{x+1} = 0.171$ and $q_{x+3} = 0.200$.
Calculate $q_{x+1} + q_{x+2}$.

y	l_y	d_y
$x+1$	1000	$1000 - 950 = 50$
$x+2$	$855 + 95 = 950$	$1000 \times 0.095 = 95$
$x+3$	$171/0.2 = 855$	$1000 \times 0.171 = 171$

$$q_{x+1} + q_{x+2} = \frac{50}{1000} + \frac{95}{950} = \boxed{0.15}$$

Exercise 3

You are given the following:

A. The probability that a person age 20 will survive 30 years is 0.7.
B. The probability that a person age 40 will survive 5 years and die in the following 5 years is 0.0475.
C. The probability that a person age 20 will die between exact ages 40 and 45 is 0.04.

Calculate the probability that a person age 20 will survive 25 years.

Restate the problem using actuarial notation:

A. ${}_{30}p_{20} = 0.7$
B. ${}_{5|5}q_{40} = 0.0475$
C. ${}_{20|5}q_{20} = 0.04$

Calculate ${}_{25}p_{20}$.

Exercise 3 scratch paper

Given:

A. ${}_{30}p_{20} = 0.7$

B. ${}_{5|5}q_{40} = 0.0475$

C. ${}_{20|5}q_{20} = 0.04$

Calculate ${}_{25}p_{20}$.

Exercise 3 solution

Given:

A. ${}_{30}p_{20} = 0.7$

B. ${}_{5|5}q_{40} = 0.0475$

C. ${}_{20|5}q_{20} = 0.04$

Calculate ${}_{25}p_{20}$.

$\star \quad l_{40} - 40 = 700 + l_{40}(0.0475)$

$l_{40} = 777$

$l_{45} = 777 - 40 = 737$

${}_{25}p_{20} = \frac{737}{1000} = \boxed{0.737}$

x	l_x	${}_5d_x$
20	1000	
$\vdots$		
40	l_{40}	$1000 \times 0.04 = 40$
45	$\star$	$l_{40}(0.0475)$
50	$1000 \times 0.7 = 700$	