The Infinite Actuary Exam STAM Online Course
A.5.2. Practice Problems on Poisson-Gamma Mixtures

1. An actuary for an automobile insurance company determines that the distribution of the annual number of claims for an insured chosen at random is modeled by the negative binomial distribution with mean 0.2 and variance 0.3.

   The number of claims for each individual insured has a Poisson distribution and the means of these Poisson distributions are gamma distributed over the population of insureds.

   Calculate the variance of this gamma distribution.

   A. 0.10 B. 0.15 C. 0.20 D. 0.25 E. 0.30

2. [3-CAS.F04.21] The number of auto claims for a group of 1,000 insured drivers has a negative binomial distribution with $\beta = 0.5$ and $r = 5$.

   Determine the parameters β and r for the distribution of the number of auto claims for a group of 2,500 such individuals.

   A. $\beta = 1.25$ and $r = 5$
   B. $\beta = 0.20$ and $r = 5$
   C. $\beta = 0.50$ and $r = 5$
   D. $\beta = 0.20$ and $r = 12.5$
   E. $\beta = 0.50$ and $r = 12.5$

3. [3.F02.5] Actuaries have modeled auto windshield claim frequencies. They have concluded that the number of windshield claims filed per year per driver follows the Poisson distribution with parameter λ, where λ follows the gamma distribution with mean 3 and variance 3. Calculate the probability that a driver selected at random will file no more than 1 windshield claim next year.

A. 0.15 B. 0.19 C. 0.20 D. 0.24 E. 0.31

4. [3.S00.4] You are given:

   (i) The claim count N has a Poisson distribution with mean L .

   (ii) L has a gamma distribution with mean 1 and variance 2.

   Calculate the probability that $N = 1$.

   A. 0.19 B. 0.24 C. 0.31 D. 0.34 E. 0.37

5. [3.Sample.12] The annual number of accidents for an individual driver has a Poisson distribution with mean λ. The Poisson means, λ, of a heterogeneous population of drivers has a gamma distribution with mean 0.1 and variance 0.01.

   Calculate the probability that a driver selected at random from the population will have 2 or more accidents in one year.

   A. 1/121 B. 1/110 C. 1/100 D. 1/90 E. 1/81

6. [4B.F96.26] You are given the following:

   - The probability that a single insured will produce 0 claims during the next exposure period is $e^{-\lambda}$
   - λ varies by insured and follows a distribution with density function $f(\lambda) = 36\lambda e^{-6\lambda}, 0 < \lambda < \infty$

   Determine the probability that a randomly selected insured will produce 0 claims during the next exposure period.

   A. Less than 0.72
   B. At least 0.72, but less than 0.77
   C. At least 0.77, but less than 0.82
   D. At least 0.82, but less than 0.87
   E. At least 0.87

7. [3.S05.10] Low Risk Insurance Company provides liability coverage to a population of 1,000 private passenger automobile drivers. The number of claims during a given year from this population is Poisson distributed.

   If a driver is selected at random from the population, his expected number of claims per year is a random variable with a Gamma distribution such that $\alpha = 2$ and $\theta = 1$.

   Calculate the probability that a driver selected at random will not have a claim during the year.

   A. 11.1% B. 13.5% C. 25.0% D. 33.3% E. 50.0%

8. [3.S00.4] You are given:

(i) The claim count N has a Poisson distribution with mean Λ.

(ii) Λ has a gamma distribution with mean 1 and variance 2.

Calculate the unconditional probability that $N = 1$.

A. 0.19 B. 0.24 C. 0.31 D. 0.34 E. 0.37

9. [3.S01.15] An actuary for an automobile insurance company determines that the distribution of the annual number of claims for an insured chosen at random is modeled by the negative binomial distribution with mean 0.2 and variance 0.4.

The number of claims for each individual insured has a Poisson distribution and the means of these Poisson distributions are gamma distributed over the population of insureds.

Calculate the variance of this gamma distribution.

A. 0.20 B. 0.25 C. 0.30 D. 0.35 E. 0.40