

The Infinite Actuary Exam STAM Online Course

C.4.7. Score Based Model Choices

Last updated April 11, 2018

1. [C.F06.22] Five models are fitted to a sample of $n = 260$ observations with the following results:

Model	Number of Parameters	Loglikelihood
I	1	-414
II	2	-412
III	3	-411
IV	4	-409
V	6	-409

Determine the model favored by the Schwarz Bayesian criterion.

- A. I B. II C. III D. IV E. V
-

The SBC means we take the loglikelihood and subtract off $(r/2) \ln(n) = 2.78r$ where r is the number of parameters. This yields

Model	r	Loglikelihood	SBC
I	1	-414	-416.8
II	2	-412	-417.6
III	3	-411	-419.3
IV	4	-409	-420.1
V	6	-409	-425.7

The SBC score is maximized in Model I, so the answer is A

2. Variant on [C.F06.22] Five models are fitted to a sample of $n = 260$ observations with the following results:

Model	Number of Parameters	Loglikelihood
I	1	-414
II	2	-412
III	3	-411
IV	4	-409
V	6	-409

Determine the model favored by the Akaike Information criterion.

- A. I B. II C. III D. IV E. V
-

A question like this just calculating which model the AIC prefers is probably too easy for the exam. The AIC means we take the loglikelihood and subtract off the number of parameters. This yields

Model	r	Loglikelihood	AIC
I	1	-414	-415
II	2	-412	-414
III	3	-411	-414
IV	4	-409	-413
V	6	-409	-415

The AIC score is maximized in Model IV, so the answer is D

3. Five models are fitted to a sample of n observations with the following results:

Model	Number of Parameters	Loglikelihood
I	1	-235.9
II	2	-233.4
III	3	-231.8
IV	4	-229.5
V	6	-228.1

What is the largest n for which the Schwarz-Bayes and Akaike Information Criteria would choose the same model?

- A. 4 B. 5 C. 49 D. 71 E. 99
-

First, let's figure out which model the AIC prefers. The AIC means we take the loglikelihood and subtract off the number of parameters. This yields

Model	r	Loglikelihood	AIC
I	1	-235.9	-236.9
II	2	-233.4	-235.4
III	3	-231.8	-234.8
IV	4	-229.5	-233.5
V	6	-228.1	-234.1

This is largest (least negative) for model IV, so that is what the AIC prefers. As n increases, the SBC will have a larger penalty for high parameters than the AIC, so there will be some n where both will choose model IV, and then the SBC will switch to something with fewer parameters. One way to see how large n can be and still prefer the same model is to check how large n needs to be to switch to each one of the other models individually.

The SBC prefers IV to III if

$$\begin{aligned}
 -229.5 - \frac{4}{2} \ln(n) &> -231.8 - \frac{3}{2} \ln(n) \\
 231.8 - 229.5 &> \frac{1}{2} \ln(n) \\
 99.5 &> n
 \end{aligned}$$

so if models I and II didn't exist, the SBC would prefer IV for n as large as 99 and then switch to III when n hits 100. But Models I and II do exist, and SBC switches away from IV earlier, namely to model II when

$$-229.5 - \frac{4}{2} \ln(n) > -233.4 - \frac{2}{2} \ln(n)$$

$$233.4 - 229.5 > \frac{1}{2} \ln(n)$$

$$49.4 > n$$

So IV is only preferred up to $n = 49$ then we switch to II at $n = 50$. To confirm that this is the answer, we can check when we prefer I to IV, but that doesn't happen until $n = 72$ (and even then it turns out the SBC of II is still higher and is preferred up to $n = 140$).

For very small n , the SBC may (and here does) prefer V, but we don't care as it will switch from V to IV as n increases and we want to know when we switch from IV to something else.

4. An actuary is trying to choose between which of five models to use. First she fit the parameters using the MLE based on a sample of $n = 130$ observations with the following results:

Model	Number of Parameters	Loglikelihood
I	1	-355.8
II	2	-353.9
III	3	-350.6
IV	4	-349.1
V	6	-347.6

Which models are preferred by the Schwarz-Bayes (SBC) and Akaike Information (AIC) criteria?

- A. SBC prefers I, AIC prefers IV B. SBC prefers IV, AIC prefers I C. SBC prefers III, AIC prefers IV
D. SBC prefers IV, AIC prefers III E. Both prefer IV

The AIC score is $\ell - r$ while the SBC is $\ell - (r/2) \ln(n)$, where r is the number of parameters, and $n = 130$. Expanding our table to include those gives

Model	r	Loglikelihood	AIC	SBC
I	1	-355.8	-356.8	-358.2
II	2	-353.9	-355.9	-358.8
III	3	-350.6	-353.6	-357.9
IV	4	-349.1	-353.1	-358.8
V	6	-347.6	-353.6	-362.2

And from those, we see the AIC is maximized by IV and the SBC maximized by III, making the answer

C