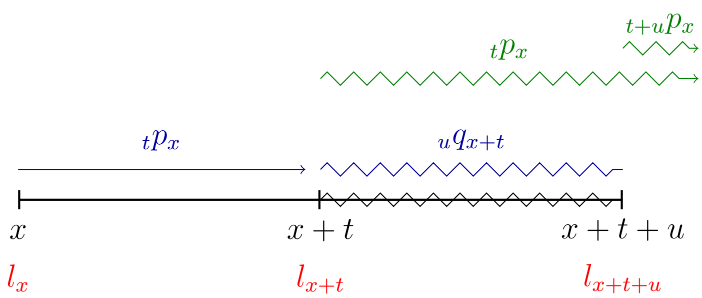
Last updated May 25, 2016.

1. [First Pass] Which of the following are true?

    1. ${}_{t|u}q_x = {}_tp_x \cdot \ {}_uq_{x+t}$
    2. ${}_{t|u}q_x = \frac{l_{x+t+u} - l_{x+t}}{l_x}$
    3. ${}_{t|u}q_x = {}_tp_x - {}_{t+u}p_x$

A. 1 B. 2 C. 3 D. 1, 2 E. 1, 3

Draw a picture for ${}_{t|u}q_x$



(i) ${}_{t|u}q_x = {}_tp_x \cdot {}_uq_{x+t}$ ✓

(ii) ${}_{t|u}q_x = \frac{l_{x+t} - l_{x+t+u}}{l_x} \neq \frac{l_{x+t+u} - l_{x+t}}{l_x}$

(iii) ${}_{t|u}q_x = {}_tp_x - {}_{t+u}p_x$ ✓

Thus, $\boxed{1, 3}$ are true.

2. [First Pass] Given that a life aged 50 will live to age 60, what is the probability p that he will die between ages 70 and 80?

Age	l_x
50	89,509
60	81,881
70	66,162
80	39,144

A. Less than 0.310
B. At least 0.310, but less than 0.315
C. At least 0.315, but less than 0.320
D. At least 0.320, but less than 0.325
E. At least 0.325

$$p = \frac{l_{70} - l_{80}}{l_{60}} = \frac{66{,}162 - 39{,}144}{81{,}881} = \boxed{0.33}$$

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3. [First Pass] You are given the following mortality table:

Age(x)	q_x	l_x	d_x
20		30,000	1,200
21			
22		27,350	
23	0.0700		
24	0.0790	23,900	

Determine the probability that a life aged 21 will die within two years.

A. Less than 0.0960
B. At least 0.0960, but less than 0.1010
C. At least 0.1010, but less than 0.1060
D. At least 0.1060, but less than 0.1110
E. At least 0.1110

$$_2q_{21} = \frac{l_{21} - l_{23}}{l_{21}}$$
$$l_{21} = l_{20} - d_{20} = 30000 - 1200 = 28800$$

$$\begin{aligned} l_{23} \times p_{23} &= l_{24} \\ l_{23} &= \frac{23900}{0.93} = 25699 \\ {}_2q_{21} &= \frac{28800 - 25699}{28800} = \boxed{0.1077} \end{aligned}$$

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4. [First Pass] Given the following portion of a life table:

x	l_x	d_x	p_x	q_x
0	1,000		0.875	
1				
2	750			0.25
3				
4				
5	200	120		
6				
7		20		1.00

Determine the value of $p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot q_6$.

A. Less than 0.055
B. At least 0.055, but less than 0.065
C. At least 0.065, but less than 0.075
D. At least 0.075
E. The answer cannot be determined from the given information.

First simplify the expression

$$p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot q_6 = {}_5p_1 \cdot q_6 = {}_{5|1}q_1 = \frac{l_6 - l_7}{l_1}$$

Now find l_1, l_6 and l_7

$$\begin{aligned} l_1 &= 1000(0.875) = 875 \\ l_6 &= 200 - 120 = 80 \\ l_7 &= 20 \end{aligned}$$

Plugging these back into the first equation we have

$$p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot q_6 = \frac{80 - 20}{875} = \boxed{0.06857}$$

5. [First Pass] You are given $S_0(x) = \dfrac{1}{1+x}$.

Determine the median future lifetime of (y).

A. $y+1$ B. y C. 1 D. $\frac{1}{y}$ E. $\frac{1}{1+y}$

Let m be the median

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\begin{aligned}
{}_m p_y &= \frac{S_0(y+m)}{S_0(y)} \\
0.5 &= \frac{\frac{1}{1+y+m}}{\frac{1}{1+y}} \\
0.5 &= \frac{1+y}{1+y+m} \\
2(1+y) &= 1+y+m \\
\boxed{1+y} &= m
\end{aligned}
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6. You are given the following information:

(i) $l_1 = 9700$

(ii) $q_1 = q_2 = 0.020$

(iii) $q_4 = 0.026$

(iv) $d_3 = 232$

Determine the expected number of survivors to age 5.

A. Less than 8,845
B. At least 8,845, but less than 8,850
C. At least 8,850, but less than 8,855
D. At least 8,855, but less than 8,860
E. At least 8,860

$$\begin{aligned}
l_2 &= l_1\, p_1 = 9700(1-.02) = 9506 \\
l_3 &= l_2\, p_2 = 9506(1-.02) = 9315.88 \\
l_4 &= l_3 - d_3 = 9315.88 - 232 = 9083.88 \\
l_5 &= l_4\, p_4 = 9083.88(1-.026) = \boxed{8847.70}
\end{aligned}$$

7. You are given the following information:

(i) The probability that two 70-year-olds are both alive in 20 years is 16%.

(ii) The probability that two 80-year-olds are both alive in 20 years is 1%.

(iii) There is an 8% chance of a 70-year-old living 30 years.

(iv) All lives are independent and have the same expected mortality.

Determine the probability of an 80-year-old living 10 years.

A. Less than 0.35
B. At least 0.35, but less than 0.45
C. At least 0.45, but less than 0.55
D. At least 0.55, but less than 0.65
E. At least 0.65

Set $l_{70} = 100$ (arbitrary). We are given:

(i) $l_{90} = l_{70}\sqrt{0.16} = 100(0.4) = 40$

(ii) $l_{100} = l_{80}\sqrt{0.01} = 0.1\, l_{80}$

(iii) $l_{100} = l_{70}(0.08) = 100(0.08) = 8$

Combining 2 and 3 we have

$$\begin{aligned} 0.1\, l_{80} &= 8 \\ l_{80} &= 80 \end{aligned}$$

Finally

$${}_{10}p_{80} = \frac{l_{90}}{l_{80}} = \frac{40}{80} = \boxed{0.5}$$

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8. Light bulbs burn out according to the following life table:

x	l_x
0	1,000,000
1	800,000
2	600,000
3	300,000
4	0

A new plant has 2,500 light bulbs. Burned out light bulbs are replaced with new light bulbs at the end of each year.

What is the expected number of new light bulbs that will be needed at the end of year 3?

A. Less than 800
B. At least 800, but less than 860
C. At least 860, but less than 920
D. At least 920, but less than 980
E. At least 980

Completing the table we have

x	l_x	d_x	$_{x\mid}q_0$
0	1,000,000	200,000	$\frac{200{,}000}{1{,}000{,}000} = 0.2$
1	800,000	200,000	$\frac{200{,}000}{1{,}000{,}000} = 0.2$
2	600,000	300,000	$\frac{300{,}000}{1{,}000{,}000} = 0.3$
3	300,000	300,000	$\frac{300{,}000}{1{,}000{,}000} = 0.3$

So of the original 2,500 bulbs 20% burn out during first year, 20% during second year and 30% during the third year.

Year 1	Year 2	Year 3
2,500(0.2) = 500	2,500(0.2) = 500	2,500(0.3) = 750

Now we need to account for the 500 bulbs that burned out during the first year. 20% of these burn out during year 2 (one year later) and 20% burn out during year 3 (two years later)

Year 1	Year 2	Year 3
2,500(0.2) = 500	2,500(0.2) = 500 500(0.2) = 100	2,500(0.3) = 750 500(0.2) = 100

Now we account for the $500 + 100 = 600$ bulbs that burned out during year 2. 20% of those will burn out during the third year (one year later). So our final table looks like

Year 1	Year 2	Year 3
2,500(0.2) = 500	2,500(0.2) = 500 500(0.2) = 100	2,500(0.3) = 750 500(0.2) = 100 600(0.2) = 120

So the total number of bulbs replaced during the 3rd year $= 750 + 100 + 120 = 970$.

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9. The graph of a piecewise linear survival function, $S_0(t)$, consists of 3 line segments with endpoints $(0, 1)$, $(25, 0.50)$, $(75, 0.40)$, $(100, 0)$.

   Calculate $\frac{{}_{20|55}q_{15}}{{}_{55}q_{35}}$.

   A. 0.69 B. 0.71 C. 0.73 D. 0.75 E. 0.77

Build a life table

x	ℓ_x
0	100 (arbitrary)
25	100(0.5) = 50
75	100(0.4) = 40
100	0

$$\frac{{}_{20|55}q_{15}}{{}_{55}q_{35}} = \frac{\frac{\ell_{35}-\ell_{90}}{\ell_{15}}}{\frac{\ell_{35}-\ell_{90}}{\ell_{35}}} = \frac{\ell_{35}}{\ell_{15}} = \frac{48}{70} = \boxed{0.6857}$$

where ℓ_{35} and ℓ_{70} are found using linear interpolation (because we given a piecewise linear survival function)

$$\ell_{15} = \frac{10}{25}(100) + \frac{15}{25}(50) = 70$$

$$\ell_{35} = \frac{40}{50}(50) + \frac{10}{50}(40) = 48$$

10. [3.S00.28] For a mortality study on college students:

(i) Students entered the study on their birthdays in 1963.

(ii) You have no information about mortality before birthdays in 1963.

(iii) Dick, who turned 20 in 1963, died between his 32nd and 33rd birthdays.

(iv) Jane, who turned 21 in 1963, was alive on her birthday in 1998, at which time she left the study.

(v) All lifetimes are independent.

(vi) Likelihoods are based upon the Illustrative Life Table.

Calculate the likelihood for these two students.

A. 0.00138 B. 0.00146 C. 0.00149 D. 0.00156 E. 0.00169

$$ {}_{12|}q_{20} \cdot {}_{35}p_{21} = \left(\frac{l_{32} - l_{33}}{l_{20}} \right) \left(\frac{l_{56}}{l_{21}} \right) = \boxed{0.001489} $$

11. [SOA Fall 2013 #24] The ILT Club has 4000 members all age 25 with independent future lifetimes. The mortality for each member follows the Illustrative Life Table.

Calculate the largest N, using the normal approximation, such that the probability that there are at least N survivors at age 85 is at least 90%.

A. 930 B. 950 C. 970 D. 990 E. 1010

$\boxed{B}$ Written solution coming soon. Please see video solution.