The Infinite Actuary Exam 1/P Online Seminar
Solutions to Problems on One Dimensional Derivatives

1. $\dfrac{d}{dx}e^{3x^2+5x} = \left[\dfrac{d}{dx}(3x^2+5x)\right]\cdot e^{3x^2+5x} = (6x+5)e^{3x^2+5x}$

2. $\dfrac{d}{dx}2^{2x} = \dfrac{d}{dx}e^{2x\ln 2} = (2\ln 2)e^{2x\ln 2} = 2\ln 2\left(2^{2x}\right)$

3. $\dfrac{d}{dx}(x^2+5)e^{2x-3} = 2xe^{2x-3} + \left(x^2+5\right)2e^{2x-3} = \left(2x^2+2x+10\right)e^{2x-3}$

4. $\dfrac{d}{dx}\dfrac{x^2}{1+x^2} = \dfrac{d}{dx}\left(1-\dfrac{1}{1+x^2}\right) = \dfrac{2x}{(1+x^2)^2}$
   Or: $\dfrac{d}{dx}\dfrac{x^2}{1+x^2} = 2x\cdot\dfrac{1}{1+x^2} + x^2\cdot\dfrac{-2x}{\left(1+x^2\right)^2} = \dfrac{(2x+2x^3)-2x^3}{\left(1+x^2\right)^2} = \dfrac{2x}{\left(1+x^2\right)^2}$

5. $\dfrac{d}{dx}\left.\dfrac{x}{1+x^2}\right|_{x=0} = \left.\dfrac{1}{1+x^2}\right|_{x=0} + x\left.\dfrac{d}{dx}\dfrac{1}{1+x^2}\right|_{x=0} = 1+0=1$

6. $\dfrac{d^2}{dx^2}\left.\dfrac{1}{1+x^2}\right|_{x=0} = \dfrac{d}{dx}\left.\dfrac{-2x}{(1+x^2)^2}\right|_{x=0} = \left.\dfrac{-2}{(1+x^2)^2}\right|_{x=0} - 2x\left.\dfrac{d}{dx}\dfrac{1}{(1+x^2)^2}\right|_{x=0} = -2-0=-2$
   Part of the point of the last two problems is that when we are evaluating our derivative at 0 and have $x\cdot$something, then we can save time by not evaluating the missing piece.

7. $\dfrac{d}{dx}\left(1-3e^{-3x}\right) = -3\cdot(-3)\cdot e^{-3x} = 9e^{-3x}$

8. $\dfrac{d}{dx}\dfrac{x}{(1+x)^2} = \dfrac{1}{(1+x)^2} - \dfrac{2x}{(1+x)^3} = \dfrac{1-x}{(1+x)^3}$

9. $\dfrac{d}{dx}\dfrac{3x-5}{(4+x)^3} = \dfrac{3}{(4+x)^3} + \dfrac{(3x-5)\cdot(-3)}{(4+x)^4} = \dfrac{27-6x}{(4+x)^4}$

10. $\dfrac{d}{dt}e^{5e^t-5} = \left(\dfrac{d}{dt}5e^t-5\right)e^{5e^t-5} = 5e^te^{5e^t-5} = 5e^{5e^t+t-5}$

11. $\dfrac{d}{dt}e^{5e^t-5-t^2} = \left(\dfrac{d}{dt}5e^t-5-t^2\right)e^{5e^t-5-t^2} = \left(5e^t-2t\right)e^{5e^t-5-t^2}$

12. $\dfrac{d}{dt}(\sin t+1) = \cos t$

13. $\dfrac{d}{dt}\dfrac{5}{t^2} = -\dfrac{10}{t^3}$

14. $\dfrac{d}{dt}\dfrac{5}{(t+1)^3} = -\dfrac{15}{(t+1)^4}$

15. $\dfrac{d}{dx}|x-2| = \begin{cases}\dfrac{d}{dx}[x-2] & x>2\\ \dfrac{d}{dx}[-(x-2)] & x<2\end{cases} = \begin{cases}1 & x>2\\ -1 & x<2\end{cases}$

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16. $\dfrac{d}{dx}|x|^3 = \begin{cases} \dfrac{d}{dx}x^3 & x>0 \\ \dfrac{d}{dx}\left(-x^3\right) & x<0 \end{cases} = \begin{cases} 3x^2 & x>0 \\ -3x^2 & x<0 \end{cases}$

17. $\dfrac{d}{dx}e^{|x+2|} = \begin{cases} \dfrac{d}{dx}e^{x+2} & x>-2 \\ \dfrac{d}{dx}e^{-(x+2)} & x<-2 \end{cases} = \begin{cases} e^{x+2} & x>-2 \\ -e^{-x-2} & x<-2 \end{cases}$

18. $\dfrac{d}{dt}e^{e^{3t}-1} = \left[\dfrac{d}{dt}\left(e^{3t}-1\right)\right]e^{e^{3t}-1} = 3e^{3t}\cdot e^{e^{3t}-1}$

19. $\dfrac{d^2}{dt^2}e^{e^{3t}-1} = \dfrac{d}{dt}3e^{3t}e^{e^{3t}-1} = 9e^{3t}\cdot e^{e^{3t}-1} + 3e^{3t}\cdot 3e^{3t}\cdot e^{e^{3t}-1}$

20. $\dfrac{d}{dt}\dfrac{1}{(1-3t)^4} = \dfrac{(-4)(-3)}{(1-3t)^5} = \dfrac{12}{(1-3t)^5}$

21. $\dfrac{d}{dt}\dfrac{1}{\left(1-\frac{t}{2}\right)^2} = \dfrac{(-2)(\frac{-1}{2})}{\left(1-\frac{t}{2}\right)^3} = \dfrac{1}{\left(1-\frac{t}{2}\right)^3}$

22. $\dfrac{d}{dx}e^{2x^2-5x+3} = \dfrac{d}{dx}\left[2x^2-5x+3\right]\cdot e^{2x^2-5x+3} = (4x-5)e^{2x^2-5x+3}$

23. $\dfrac{d^2}{dx^2}e^{2x^2-5x+3} = \dfrac{d}{dx}(4x-5)e^{2x^2-5x+3} = 4e^{2x^2-5x+3} + (4x-5)^2e^{2x^2-5x+3}$

24. $\dfrac{d}{dx}\left(2x^5+x^3+8x\right)^4 = 4\left(2x^5+x^3+8x\right)^3\left(10x^4+3x^2+8\right)$

25. $\dfrac{d}{dy}\dfrac{y^2}{6}e^{-3y} = \dfrac{y}{3}e^{-3y} - \dfrac{y^2}{2}e^{-3y}$

26. $$\begin{aligned}\frac{d^2}{dy^2}\frac{y^2}{6}e^{-3y} &= \frac{d}{dy}\left[\frac{y}{3}e^{-3y}-\frac{y^2}{2}e^{-3y}\right] = \left(\frac{1}{3}e^{-3y}-ye^{-3y}\right) - \left(ye^{-3y}-\frac{3y^2}{2}e^{-3y}\right) \\ &= \frac{1}{3}e^{-3y} - 2ye^{-3y} + \frac{3y^2}{2}e^{-3y}\end{aligned}$$

27. $\dfrac{d}{dx}\left(3x^2+4\right)^5 = 5\left(3x^2+4\right)^4\cdot 6x$

28. $\dfrac{d}{dt}\left(2t^2+1\right)\cdot e^{5t} = \left(2t^2+1\right)\cdot 5e^{5t} + 4t\cdot e^{5t}$

29. $\dfrac{d}{dx}\dfrac{e^{-x}}{x} = e^{-x}\cdot\dfrac{-1}{x^2} + \dfrac{-e^{-x}}{x}$

30. $\dfrac{d}{dz}\ln(z) = \dfrac{1}{z}$

31. $\dfrac{d}{dy}\ln(2y+5) = \dfrac{2}{2y+5}$

32. $\dfrac{d}{dt}\ln\left(t^2\right) = \dfrac{2t}{t^2} = \dfrac{2}{t}$. Or you could simplify: $\ln\left(t^2\right) = 2\ln(t)$, so $\dfrac{d}{dt}\ln\left(t^2\right) = \dfrac{d}{dt}2\ln t = \dfrac{2}{t}$

33. $\dfrac{d}{dx}e^{-x^2/2} = e^{-x^2/2}\cdot\dfrac{d}{dx}\left(\dfrac{-x^2}{2}\right) = e^{-x^2/2}\cdot(-x)$

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34. $\dfrac{d}{dx}\dfrac{x^3+5}{(2+x^3)^4} = (x^3+5)\cdot\dfrac{-4\cdot 3x^2}{(2+x^3)^5} + \dfrac{3x^2}{(2+x^3)^4}$

35. $\dfrac{d}{dx}\dfrac{2x+e^x+3}{e^{2x}-x} = (2x+e^x+3)\cdot\dfrac{\left(2e^{2x}-1\right)\cdot(-1)}{\left(e^{2x}-x\right)^2} + \dfrac{2+e^x}{e^{2x}-x}$

36. $\dfrac{d}{ds}s^3\cdot(3s^2-4s+5)^4 = 3s^2\cdot(3s^2-4s+5)^4 + s^3\cdot 4(3s^2-4s+5)^3\cdot(6s-4)$

37. $\dfrac{d}{dt}(2t^3-1)^2\cdot(3t+7)^4 = (2t^3-1)^2\cdot 4\cdot 3\cdot(3t+7)^3 + 2\cdot 6t^2\cdot(2t^3-1)\cdot(3t+7)^4$

38. $\dfrac{d}{dx}\dfrac{(x^3+1)^4}{(3x^2-5)^7} = (x^3+1)^4\cdot\dfrac{-7\cdot 6x}{(3x^2-5)^8} + \dfrac{4(x^3+1)^3\cdot 3x^2}{(3x^2-5)^7}$

39. $\dfrac{d}{dz}\left[2z-(z^5+z^3)^6\right]^3 = 3\left[2z-(z^5+z^3)^6\right]^2\left[2-6(z^5+z^3)^5(5z^4+3z^2)\right]$

40. $\dfrac{d}{dr}2r\cdot e^{-2r^2-5r} = 2r\cdot(-4r-5)\,e^{-2r^2-5r} + 2e^{-2r^2-5r}$