The Infinite Actuary Exam 1/P Online Seminar
Practice Problems on Integration by Parts

Note: I have given the anti-derivative and answer and, when easy, an intermediate step using

$$\int_0^\infty x^a e^{-bx} = \frac{a!}{b^{a+1}}$$

1. $\displaystyle\int_0^\infty xe^{-x^2}dx = -\frac{1}{2}e^{-x^2}\bigg|_0^\infty = \frac{1}{2}$ by letting $u = x^2$ and $du = 2xdx$.
   Note that this problem wasn't actually integration by parts. Just because we have an expression of the form (polynomial) $\cdot\, e^{\text{something}}$ doesn't mean we want to use parts. If we have e^{x^2} then we usually want to substitute $u = x^2$ and $du = 2x\,dx$.
2. $\displaystyle\int_0^\infty ye^{-y}dy = -ye^{-y} - e^{-y}\bigg|_0^\infty = \frac{1!}{1^2} = 1$
3. $\displaystyle\int_0^\infty 3te^{-t/2}dt = -6te^{-t/2} - 12e^{-t/2}\bigg|_0^\infty = 3\cdot\frac{1!}{(1/2)^2} = 12$
4. $\displaystyle\int_0^\infty x^2e^{-x}dx = \left(-x^2 - 2x - 2\right)e^{-x}\bigg|_0^\infty = \frac{2!}{1^3} = 2$
5. $\displaystyle\int_1^\infty 3xe^{-2x^2}dx = -\frac{3}{4}e^{-2x^2}\bigg|_1^\infty = \frac{3}{4}e^{-2}$
   Again, we have e^{x^2} so we used the substitution $u = x^2$ and $du = 2x\,dx$ to do the integral.
6. $\displaystyle\int_2^\infty 3ye^{-2y}dy = \left(-\frac{3}{2}y - \frac{3}{4}\right)e^{-2y}\bigg|_2^\infty = \frac{15}{4}e^{-4}$
7. $\displaystyle\int_3^\infty 3z^2e^{-4z}dz = \left(-\frac{3}{4}z^2 - \frac{3}{8}z - \frac{3}{32}\right)e^{-4z}\bigg|_3^\infty = \frac{255}{32}e^{-12}$
8. $\displaystyle\int_0^\infty 2re^{-r^2/3}dr = -3e^{-r^2/3}\bigg|_0^\infty = 3$ where we again substituted $u = r^2$ to do the integral instead of using parts.
9. $$\begin{aligned}\int_0^\infty (s+3)e^{-s/2}ds &= (s+3)(-2)e^{-s/2} - 1\cdot(-2)^2e^{-s/2}\bigg|_0^\infty = \left(-2s - 10\right)e^{-s/2}\bigg|_0^\infty \\ &= \frac{1!}{(1/2)^2} + \frac{3}{1/2} = 10\end{aligned}$$
10. $$\begin{aligned}\int_0^\infty (t^2+t)e^{-t/3}dt &= \left((t^2+t)(-3)e^{-t/3} - (2t+1)(-3)^2e^{-t/3} + 2(-3)^3e^{-t/3}\right)\bigg|_0^\infty \\ &= \frac{2!}{(1/3)^3} + \frac{1!}{(1/3)^2} = 63\end{aligned}$$
11. $\displaystyle\int_0^3 ye^{y}dy = ye^{y} - e^{y}\bigg|_0^3 = 2e^3 + 1$
12. $\displaystyle\int_0^{10} (x+1)e^{-2x}dx = \left((x+1)(-1/2)e^{-2x} - (-1/2)^2e^{-2x}\right)\Big|_0^1 0 = -5.75e^{-20} + .75$

13. $\displaystyle\int_0^{\pi/2} x\cos x dx = x\sin x + \cos x\Big|_0^{\pi/2} = \frac{\pi}{2} - 1$

14. $\displaystyle\int_0^{\pi} t^2\cos(t/2)dt = 2t^2\sin(t/2) + 8t\cos(t/2) - 16\sin(t/2)\big|_0^{\pi} = 2\pi^2 - 16$

15. $\displaystyle\int_1^5 3\ln(2x)dx = [3x\ln(2x) - 3x]\Big|_1^5 = [15\ln(10) - 15] - [3\ln(2) - 3]$

To do the integral, we can use parts with $u = \ln(2x)$ so $du = dx/x$, and $dv = 3dx$ so $v = 3x$. Then $uv = 3x\ln(2x)$ and $vdu = 3dx$. The point is we want $\ln(2x)$ as u because we know how to differentiate it, but not necessarily how to integrate it.