The Infinite Actuary Exam 1/P Online Seminar
Practice Problems on One Dimensional Integrals

When doing integrals, remember that derivatives and integrals are inverses. That means that you can always check your work by differentiating your answer. Also, if you don't follow where all of the constants in an answer come from, differentiating the answer will help you see where things come from.

1. $\displaystyle\int \frac{1}{(5+t)^3}dt = \frac{-1}{2}\cdot\frac{1}{(5+t)^2} + C$

2. $\displaystyle\int \frac{1}{(4+3t)^4}dt = \frac{-1}{9}\cdot\frac{1}{(4+3t)^3} + C$

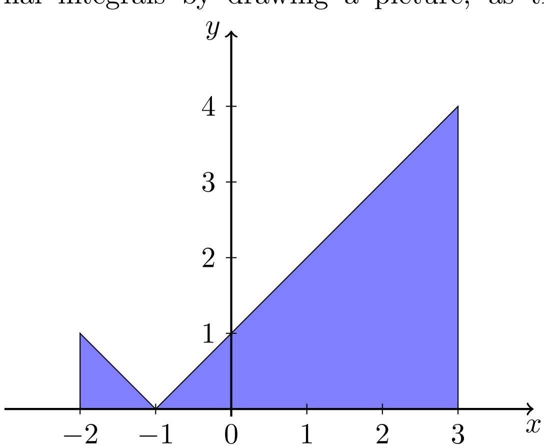
3. Let $u = t^3$ so $du = 3t^2dt$ and $\displaystyle\int \frac{t^2}{(1+t^3)^4}dt = \int \frac{du/3}{(1+u)^4} = \frac{-1}{9}\frac{1}{(1+u)^3} + C = \frac{-1}{9}\frac{1}{(1+t^3)^3} + C$

4. Let $u = 9+t$, so $t = u - 9$ and $dt = du$. This gives

$$\begin{aligned}
\int \frac{t^2}{(9+t)^5}dt &= \int \frac{(u-9)^2}{u^5}du \\
&= \int \left(\frac{u^2}{u^5} - 2\frac{9u}{u^5} + \frac{9^2}{u^5}\right)du = \int \left(u^{-3} - 18u^{-4} + 81u^{-5}\right)du \\
&= -\frac{1}{2}\cdot\frac{1}{u^2} + \frac{1}{3}\cdot\frac{18}{u^3} - \frac{1}{4}\cdot\frac{81}{u^4} + C \\
&= -\frac{1}{2}\cdot\frac{1}{(9+t)^2} + \frac{6}{(9+t)^3} - \frac{81}{4}\cdot\frac{1}{(9+t)^4} + C \\
&= \frac{-2t^2 - 12t - 27}{4(9+t)^4} + C
\end{aligned}$$

5. $\displaystyle\int_0^{10} \frac{1}{1+t}dt = \ln(1+t)\Big|_0^{10} = \ln(11) - \ln 1 = \ln(11)$

6. $\displaystyle\int_{-2}^{3} |x+1|dx = \int_{-2}^{-1}(-x-1)dx + \int_{-1}^{3}(x+1)dx = \frac{1}{2} + \frac{16}{2} = \frac{17}{2}$ Note: I computed the final integrals by drawing a picture, as the area under the curve is the area of two triangles.



7. $\displaystyle\int_2^7 x^2(1+x)dx = \left(\frac{1}{3}x^3 + \frac{1}{4}x^4\right)\Big|_2^7 = \left(\frac{7^3}{3} + \frac{7^4}{4}\right) - \left(\frac{2^3}{3} + \frac{2^4}{4}\right) = 707.9$

8. $\displaystyle\int_3^5 x^2 dx = \frac{5^3}{3} - \frac{3^3}{3} = \frac{98}{3}$

9. $\displaystyle\int 5(2x+1)^3 dx = \frac{5}{2\cdot 4}(2x+1)^4 + C$

10. $\displaystyle\int_5^\infty e^{-3x} dx = \left.\frac{-1}{3}e^{-3x}\right|_5^\infty = -\frac{1}{3}e^{-\infty} + \frac{1}{3}e^{-15} = \frac{1}{3}e^{-15}$

11. $\displaystyle\int xe^{-3x^2} dx = \frac{-1}{6}e^{-3x^2} + C$ by letting $u = x^2$ and $du = 2xdx$

12. $\displaystyle\int_\infty^\infty e^{-|x|} dx = \int_0^\infty e^{-x} dx + \int_{-\infty}^0 e^x dx = \left[-e^{-\infty} - (-e^0)\right] + \left[e^0 - e^{-\infty}\right] = 2$

13. $\displaystyle\int_0^2 t\cos(t^2) dt = \left.\frac{1}{2}\sin\left(t^2\right)\right|_0^2 = (1/2)\sin 4$ by substituing $u = t^2$ so $du = 2tdt$ and $tdt = du/2$.

14. $\displaystyle\int x(x^2+2)^3 dx = \frac{1}{8}(x^2+2)^4 + C$ by using $u = x^2+2$ so $xdx = du/2$.

15. $\displaystyle\int x^2(x^3 - x - 1)dx = \int \left(x^5 - x^3 - x^2\right) dx = \frac{1}{6}x^6 - \frac{1}{4}x^4 - \frac{1}{3}x^3 + C$

16. $\displaystyle\int_{30}^\infty 3\cdot\frac{30^3}{x^4}\,dx = \left.-\frac{30^3}{x^3}\right|_{30}^\infty = -\frac{30^3}{\infty^3} + \frac{30^3}{30^3} = 1$

17. $\displaystyle\int_{30}^\infty 3\cdot\frac{30^3}{x^3}\,dx = \left.-\frac{3}{2}\cdot\frac{30^3}{x^2}\right|_{30}^\infty = -\frac{3}{2}\cdot\frac{30^3}{\infty^2} + \frac{3}{2}\cdot\frac{30^3}{30^2} = 45$

18. $\displaystyle\int_{30}^\infty 3\cdot\frac{30^3}{x^2}\,dx = \left.-\frac{3}{1}\cdot\frac{30^3}{x}\right|_{30}^\infty = -\frac{3}{1}\cdot\frac{30^3}{\infty} + \frac{3}{1}\cdot\frac{30^3}{30} = 2,700$

19. $\displaystyle\int_0^\infty 3\cdot\frac{30^3}{(x+30)^4}\,dx = \left.-\frac{30^3}{(x+30)^3}\right|_0^\infty = -\frac{30^3}{(\infty+30)^3} + \frac{30^3}{(0+30)^3} = 1$

20. $\displaystyle\int_0^\infty 3\cdot\frac{30^3 x}{(x+30)^4}\,dx = \int_{30}^\infty 3\frac{30^3(u-30)}{u^4}\,du$ where $u = x+30$, giving
$\displaystyle\int_{30}^\infty \left(3\frac{30^3}{u^3} - 3\frac{30^4}{u^4}\right) du = 45 - \frac{30^4}{30^3} = 15$

21. $\displaystyle\int_0^2 \left(x^2+2\right)\left(x^3+6x\right)^5 dx = \left.\frac{1}{3}\cdot\frac{1}{6}\left(x^3+6x\right)^6\right|_0^2 = \frac{20^6}{18}$ by letting $u = x^3+6x$ and $du = 3\left(x^2+2\right)dx$.

22. $\displaystyle\int_0^4 \frac{x^2}{4}dx = \left.\frac{x^3}{4\cdot 3}\right|_0^4 = \frac{4^3}{12} - \frac{0^3}{12} = \frac{16}{3}$

23. $\displaystyle\int \frac{1}{(3+5z)^3}\,dz = \frac{-1}{2\cdot 5}\cdot\frac{1}{(3+5z)^2} + C$

24. $\displaystyle\int t^3\left(5+t^4\right)^2 dt = \frac{1}{3\cdot 4}(5+t^4)^3 + C$ by letting $u = 5+t^4, du = 4t^3dt$ and $t^3dt = du/4$

25. $\displaystyle\int (2y+1)\left(y^2+y\right)^3 dy = \frac{1}{4}(y^2+y)^4 + C$ by letting $u = y^2+y$ and $du = (2y+1)dy$

26. $\displaystyle\int \frac{5}{3+2z}dz = \frac{5}{2}\ln(3+2z)+C$

27. $\displaystyle\int \frac{y^2}{1+y^3}dy = \frac{1}{3}\ln(1+y^3)+C$ by letting $u=y^3$ and $du=3y^2dy$

28. $\displaystyle\int 4e^{5t}dt = 4e^{5t}\cdot\frac{1}{5}+C = \frac{4}{5}e^{5t}+C$

29. $\displaystyle\int_0^5 e^{4x+1}dx = \frac{1}{4}e^{4x+1}\Big|_0^5 = \frac{e^{21}-e}{4}$

30. $\displaystyle\int_{-5}^{5}|2s+1|ds = \int_{-5}^{-1/2}-(2s+1)ds+\int_{-0.5}^{5}(2s+1)ds = (-s^2-s)\Big|_{-5}^{-0.5}+(s^2+s)\Big|_{-0.5}^{5} = (-0.25+0.5)-(-25+5)+(25+5)-(0.25-0.5) = 50.5$ Alternatively, the graph is two triangles, the first has height 9 and base 4.5, while the second has height 11 and base 5.5, so the area is $9\cdot4.5\cdot0.5+11\cdot5.5\cdot0.5 = 50.5$