The Infinite Actuary Exam 4/C Online Seminar
A.2. Practice Problems on Key Continuous Distributions

1. Calculate the kurtosis for the exponential distribution with mean 10.

A. 1 B. 3 C. 6 D. 9 E. 12

2. Losses follow a 2-point mixture, that with probability 0.7 comes from an exponential distribution with mean 10, while with probability 0.3 comes from a distribution with density

$$f(x) = \begin{cases} \frac{1}{10}e^{-(x-\delta)/10} & x > \delta \\ 0 & \text{otherwise.} \end{cases}$$

If the mean loss amount is 13, what is the probability of a loss exceeding 15?

A. 0.298 B. 0.308 C. 0.318 D. 0.328 E. 0.338

3. Losses are a mixture, and with probability p come from an exponential distribution with mean 10, while with probability $1-p$ they come from a distribution with density

$$f(x) = \begin{cases} \frac{1}{10}e^{-(x-\delta)/10} & x > \delta \\ 0 & \text{otherwise.} \end{cases}$$

If the mean loss amount is 13, and the variance of losses is 136, what is p?

A. 0.4 B. 0.5 C. 0.6 D. 0.7 E. 0.8

4. Losses are a mixture of two exponentials. With probability p come from an exponential distribution with mean 10, while with probability $1-p$ they come from an exponential with mean $10+\delta$.

   If the mean loss amount is 12, and the variance of losses is 156, what is p?

   A. 0.2 B. 0.3 C. 0.4 D. 0.5 E. 0.6

5. Losses are exponential with mean 50. What is the median of those losses that exceed 100?

A. 35 B. 104 C. 135 D. 150 E. 204

6. Losses are exponential with median 50. Two losses occur in a year, with independent loss amounts. What is the probability that at least one of those losses exceeds 100?

A. 0.06 B. 0.25 C. 0.44 D. 0.56 E. 0.60

7. Losses are exponential with median 50. Two losses occur in a year, with independent loss amounts. What is the probability that both of those losses exceeds 100?

A. 0.06 B. 0.25 C. 0.44 D. 0.56 E. 0.60

8. Losses are exponential with median 50. Two losses occur in a year, with independent loss amounts. What is the probability that the sum of those losses exceeds 100?

A. 0.06 B. 0.25 C. 0.44 D. 0.56 E. 0.60

9. Suppose X has a Gamma distribution with parameters $\alpha > 1$ and θ. Find the mode of X.

A. 0 B. $(\alpha - 1)\theta$ C. $\alpha\theta$ D. $(\alpha - 1)\theta^2$ E. $\alpha\theta^2$

10. [3.F01.37] For watches produced by a certain manufacturer:

(i) Lifetimes follow a single-parameter Pareto distribution with $\alpha > 1$ and $\theta = 4$.

(ii) The expected lifetime of a watch is 8 years.

Calculate the probability that the lifetime of a watch is at least 6 years.

A. 0.44 B. 0.50 C. 0.56 D. 0.61 E. 0.67

11. [3.S06.25] Calculate the skewness of a Pareto distribution with $\alpha = 4$ and $\theta = 1{,}000$.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

12. Losses are a 2-point mixture. 30% of the time, losses come from a Pareto distribution with $\alpha = 3$ and $\theta = 10$, and 70% of the time losses come from a Pareto distribution with $\alpha = 6$ and $\theta = 10$. What is the median loss amount?

A. 1.50 B. 1.53 C. 1.57 D. 1.60 E. 1.64

13. [M.S05.9] A loss, X, follows a 2-parameter Pareto distribution with $\alpha = 2$ and unspecified parameter θ. You are given:

$$\mathrm{E}\left[X - 100 \mid X > 100\right] = \frac{5}{3}\mathrm{E}\left[X - 50 \mid X > 50\right]$$

Calculate $\mathrm{E}\left[X - 150 \mid X > 150\right]$.

A. 150 B. 175 C. 200 D. 225 E. 250

14. [M.S05.34] The distribution of a loss, X , is a two-point mixture:

(i) With probability 0.8, X has a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 100$.

(ii) With probability 0.2, X has a two-parameter Pareto distribution with $\alpha = 4$ and $\theta = 3000$.

Calculate $\Pr(X \leq 200)$.

A. 0.76 B. 0.79 C. 0.82 D. 0.85 E. 0.88

15. The average height of adult Americans is 176 cm, with a standard deviation of 6 cm, for males, and 163 cm, with a standard deviation of 5 cm, for females. If heights of each group are normally distributed, what is the probability that a randomly selected American male is taller than a randomly selected American female?

A. 0.85 B. 0.88 C. 0.91 D. 0.93 E. 0.95

16. [3.F05.32] For a certain insurance company, 60% of claims have a normal distribution with mean 5,000 and variance 1,000,000. The remaining 40% have a normal distribution with mean 4,000 and variance 1,000,000.

Calculate the probability that a randomly selected claim exceeds 6,000.

A. Less than 0.10
B. At least 0.10, but less than 0.15
C. At least 0.15, but less than 0.20
D. At least 0.20, but less than 0.25
E. At least 0.25

17. Suppose X and Y are independent normals, each with variance 1,000,000, and with $\mathrm{E}[X] = 5{,}000$ and $\mathrm{E}[Y] = 4{,}000$.

Find $\mathrm{P}[0.6X + 0.4Y > 6{,}000]$

A. 0.026 B. 0.052 C. 0.081 D. 0.104 E. 0.123

18. [1.S01.33] For Company A there is a 60% chance that no claim is made during the coming year. If one of more claims are made, the total claim amount is normally distributed with mean 10,000 and standard deviation 2,000.

    For Company B there is a 70% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 9,000 and standard deviation 2,000.

    Assume that the total claim amounts of the two companies are independent.

    What is the probability that, in the coming year, Company B's total claim amount will exceed Company A's total claim amount?

    A. 0.180 B. 0.185 C. 0.217 D. 0.223 E. 0.240

19. Loss amounts are normally distributed, with a 6.68% chance of exceeding 102 and a 2.28% chance of exceeding 105. What is the probability that a randomly selected loss is greater than 95?

A. 0.11 B. 0.37 C. 0.48 D. 0.66 E. 0.91

20. Let Y be a mixture of X_1 and X_2, where X_1 is a normal random variable with mean 0 and standard deviation 1, and X_2 is a normal random variable with mean 0 and standard deviation 5. If $\mathrm{P}[Y = X_1] = 0.9$, what is the kurtosis of Y?

    Recall that the kurtosis of a normal random variable is 3.

    A. 3.0 B. 8.2 C. 12.4 D. 16.5 E. 49.5

21. Losses are lognormal with median 3 and mean 4. What is the variance of a randomly selected loss?

A. 0.6 B. 2.7 C. 4.5 D. 7.4 E. 12.4

22. Losses are lognormal with mean 3 and standard deviation 2. What is the probability that a loss that exceeds 1 will be greater than 4?

A. 0.23 B. 0.26 C. 0.37 D. 0.86 E. 0.89

23. [4B.F97.26] You are given the following:

- In 1996, losses follow a lognormal distribution with parameters μ and σ.
- In 1997, losses follow a lognormal distribution with parameters $\mu + \ln(k)$ and σ, where $k > 1$.
- In 1996, $100p\%$ of the losses exceed the mean of the losses in 1997.

Determine σ. Note: z_p is the $100p$th percentile of a normal distribution with mean 0 and variance 1.

A. $2\ln(k)$ B. $-z_p \pm \sqrt{z_p^2 - 2\ln(k)}$ C. $z_p \pm \sqrt{z_p^2 - 2\ln(k)}$ D. $\sqrt{-z_p \pm \sqrt{z_p^2 - 2\ln(k)}}$ E. $\sqrt{z_p \pm \sqrt{z_p^2 - 2\ln(k)}}$

24. Loss amounts have a survival function given by

$$S(x) = \begin{cases} 1 & x < 0 \\ e^{-2x^2} & x \geq 0 \end{cases}$$

What is the average loss amount?

A. $\sqrt{\frac{\pi}{8}}$ B. $\sqrt{\frac{\pi}{4}}$ C. $\sqrt{\frac{\pi}{2}}$ D. $\sqrt{\pi}$ E. $\sqrt{2\pi}$

25. Loss amounts have a survival function given by

$$S(x) = \begin{cases} 1 & x < 1 \\ e^{-2x^2} & x \geq 1 \end{cases}$$

What is the average loss amount?

A. 0.03 B. 0.77 C. 1.03 D. 1.77 E. 2.41

26. The density function of a random variable is proportional to e^{-x^2} for $x \geq 0.5$ and is 0 otherwise. Find $\mathrm{P}[X \geq 1]$

A. 0.08 B. 0.33 C. 0.55 D. 0.76 E. 0.92

27. Losses are modeled with a lognormal distribution with parameters μ and σ. If the median loss amount is 1.06 and the mean loss amount is 1.08, what is σ?

A. 0.01 B. 0.02 C. 0.04 D. 0.10 E. 0.19

28. Losses are modeled with a lognormal distribution with mean 0.42 and variance 1.65. Find the probability that losses are at least 1.

A. 0.09 B. 0.18 C. 0.27 D. 0.33 E. 0.36