ERM Sample Study Manual

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Each reading (and sub-chapters if applicable) are also bookmarked in the PDF for ease of navigation in your favorite desktop, tablet, or smartphone PDF viewer. If you have additional questions about the detailed study manual, please email me.

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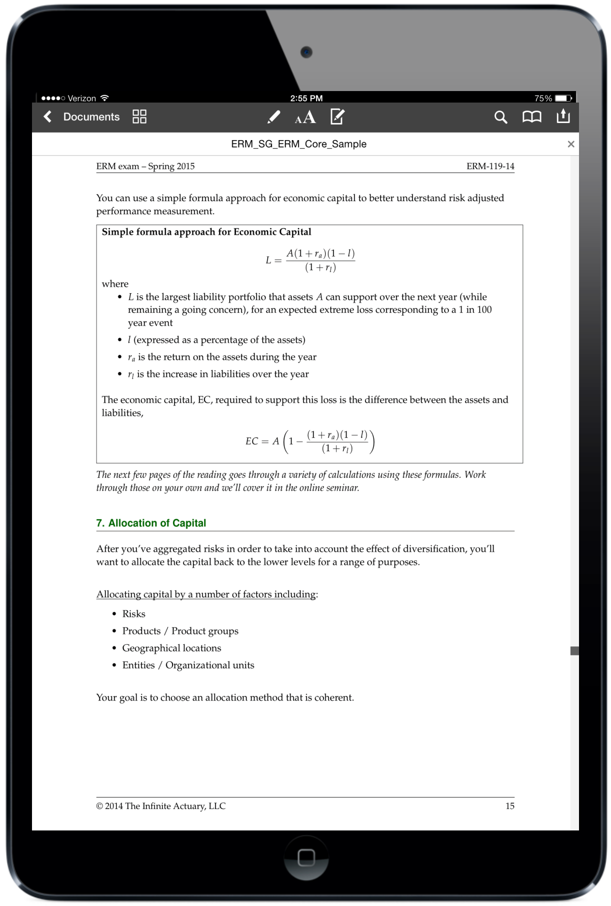

ERM exam Detailed Study Guide Sample

ERM-119-14: Allocation of Risks and Allocation of Capital

Milliman - September 2009

Overview

This article talks about risk assessment methodologies, aggregation techniques, and practical applications of capital allocation.

The key testable topics include:

- Describe the risk assessment methodologies that can be used once a specific risk has been identified
- Describe Multivariate Stress Tests
- Recommend methods for model calibration
- Compare and contrast the two top-down approaches for aggregating capital, namely correlation and copulas
- Describe the two applications of capital allocation, namely pricing and risk budgeting
- Compare, contrast and calculate the common performance measures
- Calculate economic capital using the simple formula approach
- State the three properties of a coherent allocation method
- Compare and contrast the four marginal approaches to allocating capital
- Allocate capital using the pro-rata approach and the discrete marginal contributions approach
- Describe game theory, the Shapley Value, and the Aumann-Shapley Value

Section 4. Risk Assessment Methodologies

First you have to identify the sources of the risks. Then there are various methodologies available to determine the economic capital required for each risk factor.

Methodologies to determine the economic capital required for each risk factor

- Immediate stress approach
  - Apply the risk factor stress at current time (t=0)
  - Revalue the assets and liabilities and calculate the net change
  - Exclude the receipt or payment of cash flows such as premiums received, claims paid, and interest
  - Exclude the impact of future management actions and dynamic hedging
  - Now you get an amount that represents a capital strain. Set the risk capital equal to it
  - Otherwise set to zero if no other relevant stress scenarios are considered
  - Advantage: Simple approach to calculate and understand
  - Disadvantage: Unable to capture the risks and risk mitigation impacts arising from adverse scenarios that occur over time
- Projections Scenarios approach
  - A scenario for each risk factor is postulated to occur over the shock application period
  - Treat this period as two discrete time steps (t=0 and t=n) or you may break it up into a number of smaller time steps
  - Use a larger number of time steps if you either the liabilities are path dependent or if a dynamic risk management strategy is being used
  - At each time step, calculate the value of both assets and liabilities by taking into account:
    * The value of the risk factor
    * Any dynamic interactions such as lapses and moneyness of guarantees
    * The impact of any dynamic risk management strategy
  - Calculate cash flows such as premiums, claims, and interest if you are projecting the P&L and the balance sheet
  - Set the amount of risk capital equal to the present value of the net P&L results
  - Advantages:
    * Ability to capture dynamic interactions and risk management strategies and the impact of cash flow effects
    * Realistically models the P&L and balance sheet
    * Provides more flexibility and a realistic basis for specifying adverse scenarios
    * Captures a wider range of adverse risk scenarios

- Disadvantages:
  * May require the use of nested stochastic techniques → Can be complex and computationally intensive

Multivariate Stress Tests

- You can use the two methodologies above for scenarios involving two or more risk factors at the same time
- This would generate an economic risk capital amount that relates to whatever risk factors have been included in the scenario
- The capital derived by use of a multivariate stress is not equivalent to the sum of the capital derived at the individual risk factor stress (in many situations)
  - Due to the interaction between risk factors
- If you use an immediate stress methodology, then multiple risk factors can be specified to apply instantaneously
- If you are using a projection scenario methodology, then multiple risk factors need to be modeled in the form of a projection scenario
  - You would have to use economic scenario generation models
- Advantages:
  - Allows for the generation of the full distribution of P&L results under a range of assumptions and risk management bases
  - It enables economic capital to be calculated from the end results for a range of different confidence levels
  - Can provide significant insight into the pros and cons of alternative risk management strategies and the business management issues involved across the entire shock application period and beyond
  - You can use the result of the ESGs to look at the tail for economic capital purposes and do profitability analysis
- Disadvantage: Involves greater modeling complexity and computational requirements

Calibrating single risk factor methodologies

- Calibrate each risk factor stress that is specified by the external party
- This can be cumbersome to determine these calibrations if the model is developed for internal management purposes
- Start with examining historical data
- Be conscious of the difference between the shock calibration period and the shock application period

Difference between market-consistent and real-world approaches

- Market-consistent approaches
  - Calibrate ESG parameters so that the models reproduce the market prices of the instruments in the calibration set
- Real-world approach
  - Calibrate parameters so that the model produces a distribution of risk factor results that are more aligned to expected realistic experience

Using historical events to calibrate

- You can use historical events to calibrate single or multiple factor immediate stress methodologies or single factor projection methodologies
- You can use history to help calibrate the parameters of each of the ESG models

ESG models used to model various risk factors

Risk Factor	Typical Distribution Assumed	Model
Interest Rates	Changes normally distributed; nominal rates floored at zero	Hull-White, Libor market model, Jarro-Yildlrim model for inflation and real rates. Bond returns a function of interest rates, credit spreads and duration.
Equity	Equity capital returns normally distributed	Excess returns above cash; modeled using lognormal Brownian motion. ARCH and GARCH models for dividend yields if necessary. Stable distribution of regime switching models can be used to model fat tails.
Property	Property capital returns normally distributed	Typically similar to equity models. Use of GARCH models may be appropriate to capture serial autocorrelation.
Spreads	Credit spread transition matrix, changes in credit spreads normally distributed	Jarrow-Landow-Tumbull
Currency	Currency changes normally distributed	Lognormal Brownian motion models, although potentially arbitrage free model as well.
Correlations	Typically constant	Could use regime switching model to address tail dependencies

5. Aggregation Techniques

Tips for aggregation techniques

- Do this after you've chosen a risk measure and calculated the risks
- You then aggregate risks across different products, lines, geographic areas, etc.
- Aggregated capital < sum of capital required for each risk being aggregated
- Significant interactions can exist between risks

Interaction between credit and market risk

- Credit and market risk can compound each other
- Authors suggest that top-down approaches can underestimate the total capital required to support credit and market risks
  - **Top-Down Approach**: calculating capital requirements separately for each risk and then aggregating
- Authors suggest that a bottom-up approach is more appropriate to determine capital requirements for these risks
  - **Bottom-Up Approach**: calculating the capital requirements for each contract taking capital and market risk into account together

Two top-down approaches for aggregating capital

1. Correlation
2. Copulas

Correlation

Basics of Correlation

- A measure of the strength and direction of a linear relationship between random variables
- $$Corr(X, Y) = \frac{E((X - E[X])(Y - E[Y]))}{\sigma_x \sigma_Y}$$
- Measures how two variables move relative to one another
- It is a scale invariant statistics that ranges from -1 to +1

How to interpret correlation

- **Correlation near +1**: If two variables tend to move in the same direction, regardless of the size of movement.
- **Correlation near -1**: If two variables tend to move in the opposite direction, regardless of the size of movement.
- **Correlation near 0**: If two variables tend to move in completely random ways with respect to one another, regardless of the size of movement.

Formula to aggregate risks using correlation

$$\text{Total Risk} = (\sum_i \sum_j \rho_{ij} X_i X_j)^{\frac{1}{2}}$$

- $i, j = 1, 2, ..., n$ (n is the number of risks being aggregated)
- ρ_{ij} is the correlation between risks i and j
- X_i is the risk measure output of risk i. VaR is an example.

Correlation Matrix

- Specified for the correlation between risks
- Used to calculate new totals for each row using the formula above

Considerations for correlation

- The correlation approach assumes that the risks are normally distributed
- Assumes the dependence structure can be specified via the margins of a Gaussian distribution
- The combined risk distribution is multivariate normal $\rightarrow$ Could introduce unacceptable distortion where the risks are not normally distributed
- Correlations tend to behave differently in extreme situations
- When under stress conditions, the correlation approach of aggregating risks fails

Copulas

The idea of the copula comes from Sklar's theorem.

Sklar's theorem

- Suppose $M(x)$ and $N(y)$ are the two main marginal distributions of the bivariate distribution $Z(x, y)$
- Then there exists a function, C, such that $Z(x, y) = C(M(x), N(y))$
- This function, C, is called a copula
- All continuous multivariate functions contain a unique copula

Applying a copula to economic capital

- M and N are functions of two risk distributions
- Downfall of the correlation approach
  - A risk measure is applied to each of these distributions
  - The resulting risk measure amounts, say M' and N' would be aggregated using a correlation assumptions between risks M and N in order to calculate the total capital requirement
  - The correlation approach fails because it assumes the correlation between M and N is constant for all realizations $M(x)$ and $N(y)$
- Copulas solve this problem
  - The function, C, is a plane and can be specified such that the interaction between M and N differs at different parts of each of the distributions of M and N
  - In practice, you would use transformed versions of M and N rather than their original risk distributions
  - Each risk is transformed to a uniform distribution on the interval [0,1]
  - You can use the cumulative distribution function of each risk

Steps for simulating from the marginal and aggregate risk distributions

1. Structure the copula
2. Simulate a uniform random variable for each risk distribution (e.g. p and q)
3. Calculate the cumulative distribution of each of the risk distributions at the point's p and q respectively
   - Assume the distributions $M(x)$ and $N(y)$ are the cumulative distributions of the underlying risk distributions
   - Assume $M(p)$ and $N(q)$ are calculated
4. Capture the interaction between $M(x)$ and $N(y)$ by calculating $r = C(M(p), N(q))$
5. Calculate the correlated risk realizations for each of the marginal risk distributions as the inverse of the M and N distributions.

MultiVariate Methods

Multivariate methods to calculate economic capital

- When you use multiple risk factor stresses. you get an economic capital that includes the impact of risk aggregation directly and all risk factors are included
- The benefit is that the risk factor relationships or correlations are defined explicitly with respect to the risk factors themselves
- Multiple product groups, geographic locations and entities can be simultaneously assessed to calculate an aggregate risk capital amount
- Diversification effects are modeled implicitly since any interactions between various product groups, geographic locations or entities are capture directly

6. Application of Capital Allocation

Two applicatons of capital allocation

1. Pricing
2. Risk budgeting and Capital Allocation

Pricing and Technical Provisions

Aim of allocating capital to business units, lines, and products

- To correctly allow for the cost of capital in pricing exercises
- The cost of capital is usually calculated as a product of the amount of capital allocated to a product and the return-on-capital requirement
- The target price is generally greater where the risk is more concentrated (or less diversified) as more capital is allocated to such a risk

Risks that are well-diversified are allocated less capital:

- These well-diversified risks get a lower capital charge in the pricing exercise
- Drawbacks
  - There's no unique way to allocate capital
  - A line that attracts the same amount of risk may be allocated different amounts of capital by two different insurers → Leads to two different premiums for the same risk

Other considerations for pricing:

- Some believe you that when you price product risks, you should reflect the market consistent risk premium
- Also, shareholder rewards shouldn't change due to management's method of allocating capital to individual product lines
- Capital requirements set using a market value margin approach should reflect the market price of the risk
- When you use internal solvency capital requirements to define the solvency capital required to support a line of business → You should vary the requirements according to the internal mechanics of each company

The solvency capital requirements for similar business in different companies can differ. As a result, these companies will place different market-consistent values on similar liabilities.

Approach of placing different market-consistent values on similar liabilities:

- This allows for the diversification between all non-hedgeable risks when you calculate solvency capital requirements.
- You would do this to calculate the cost of capital loadings.
- Drawback:
  - The technical provisions of an insurer are smaller than the technical provisions required to break the insurer up into business units that can be transferred to third parties
  - When financial institutions get into trouble, they need the option to unload parts of the institution to third parties to stay in business

Risk Budgeting and Capital Allocation

Risk Budgeting:

- A process where managers decide in which areas to accept risk
  - Areas include lines, products, geographical areas, etc.
- A total risk budget for the entire organization is the starting point
- Risk targets (or budgets) are set for each area
  - Usually done top-down, starting with the organization level, then with business units and lines
- Capital allocation techniques are used to allocate the total risk budget down through the different layers → Leads to a budget for each level
- Managers then compare the budget at each level to the risk amount actually taken on that level
- Managers use this comparison as a decision tool to determine which levels require additional capital or where capital may be released
- The budget can be used to allocate responsibility to managers

Using reinsurance and hedging as risk mitigation strategies:

- If done at the business unit level → The risk position of that unit and for the entire insurer will be impacted
- There will restrictions on the movement of capital

Two important concepts on the movement of capital:

1. Fungibility of capital
   - The ability of capital to absorb losses, no matter where the capital is held or where the losses occur within the group
2. Transferability of capital
   - The actual ability of a business unit to transfer assets or liabilities to the rest of the group
   - Taking into account the time and costs of the transfer

Allocating capital as a way to manage the insurer's risk appetite:

- The capital is allocated to a business unit,
- That capital amount becomes a limit for managers in that unit on the risk they are allowed to accept
- You can have disincentives to discourage exceeding the risk budget
- Risk decisions can be taken with a clearer view on how they impact the risk appetite of the total organization

Allocate capital to risk types:

- This is another approach to risk budgeting
- Useful if you want to manage the types of risk a company accepts across all its subsidiaries
- Disadvantage: This approach is less useful for management decision-making
  - Since it's unclear how to make use of the available budget for a specific risk

Silo risk management

- When line-managers do a good job of managing risks within a budget
- But don't apply integrated company-level risk principles

It is often recommended to have a sufficient buffer between economic capital and risk-taking capacity.

Two parts of a sufficient buffer:

1. A Strategic buffer
   - Use it to write new business and for taking advantage of new business opportunities
   - Use it for possible future additional regulatory requirements
   - Use the strategic ambitions of management to determine the size of this buffer
2. A Technical Buffer
   - Use it for business cycles impacts outside the one-year horizon of economic capital
   - Also use it against the volatility of the capital base
   - Analyze the company's financial condition to help determine this buffer

Risk adjusted performance measurement

Common performance measures:

- Return on Assets (ROA)
- Return on Equity (ROE)
- Risk Adjusted Return on Capital (RAROC)
- Risk Adjusted Return on Risk Adjusted Capital (RARORAC)

RAROC and RARORAC:

- Often used as performance measures
- Measures of the profitability of a portfolio
- Takes into account the risk assumed to generate the profits
- Considered more accurate and comparable than the more traditional return measures

ROE and ROA

- More traditional measures
- Focus on performance relative to accounting balance sheet items

Two drawbacks of the traditional return measures:

1. Focusing on assets ignores leverage effects
   - ROA fails here
   - ROE does capture leverage impacts
2. There is no distinction between classes or riskiness of assets
   - Neither ROE or ROA allow for risk being accepted to achieve the return generated

You may be tested to compare and contrast RAROC and RARORAC.

RAROC

- Calculated as the risk adjusted return (the long-term expected return across the insurance cycle) divided by available capital

$$\text{RAROC} = \frac{\text{Risk Adjusted Return}}{\text{Capital}}$$

- More interesting for shareholders since it is a measure of risk adjusted return on the total capital supplied by the shareholders

RARORAC:

- Calculated as risk adjusted return divided by required capital
- Required capital captures the risk assumed by the insurer in generating the numerator (the economic return), after allowing for the diversification effect between the risks at the company level

$$\text{RARORAC} = \frac{\text{Risk Adjusted Return}}{\text{Required Capital}} = \frac{\text{(Revenues - Costs - Expected Losses)}}{\text{Required Capital}}$$

- Measures the risk adjusted return against the capital required to generate it
- More suitable for internal management purposes
- Doesn't take the additional capital available between required and available capital
- Can be calculated at any level where comparison of performance is desired

You can use a simple formula approach for economic capital to better understand risk adjusted performance measurement.

Simple formula approach for Economic Capital

$$L = \frac{A(1+r_a)(1-l)}{(1+r_l)}$$

where

- L is the largest liability portfolio that assets A can support over the next year (while remaining a going concern), for an expected extreme loss corresponding to a 1 in 100 year event
- l (expressed as a percentage of the assets)
- r_a is the return on the assets during the year
- r_l is the increase in liabilities over the year

The economic capital, EC, required to support this loss is the difference between the assets and liabilities,

$$EC = A\left(1 - \frac{(1+r_a)(1-l)}{(1+r_l)}\right)$$

The next few pages of the reading goes through a variety of calculations using these formulas. Work through those on your own and we'll cover it in the online seminar.

7. Allocation of Capital

After you've aggregated risks in order to take into account the effect of diversification, you'll want to allocate the capital back to the lower levels for a range of purposes.

Allocating capital by a number of factors including:

- Risks
- Products / Product groups
- Geographical locations
- Entities / Organizational units

Your goal is to choose an allocation method that is coherent.

Properties of a coherent allocation method:

1. No undercut
   - A sub-portfolio's allocation should be no more than its standalone capital requirement
2. Symmetry
   - If the risk of two sub-portfolios (as measured by the risk measure) is the same, then the allocation should be the same for each
3. Risk-free allocation
   - Capital allocated to a risk-free line of business should be zero

Now we'll get into the two main approaches to allocate capital:

- Marginal approaches
- Game theory

Marginal Approaches

Marginal approaches to allocating capital:

- Pro-rata or linear marginal contributions
- Discrete marginal contributions
- Continuous marginal contributions
- Myers-Read allocation method

<u>Pro-rata or linear marginal contributions</u>:

- The simplest approach
- The total capital requirements (including the diversification effect) of the combined portfolio XYZ is allocated pro-rata to each of X, Y, Z.
- This approach allocates the diversification benefits across the portfolios in proportion to each portfolio's individual capital requirement
- It doesn't penalize highly correlated portfolios
- It doesn't reward those portfolios that increase the overall diversification effect

Portfolio	Pro-Rata Allocation
X	12.4
Y	6.9
Z	17.7
XYZ	37 (=12.4+6.9+17.7)

<u>Discrete marginal contributions</u>:

- First determine the capital of the total portfolio, excluding the portfolio in question
  - e.g. the capital of portfolio YZ is 25.9
- Subtract this capital from the capital of the total portfolio
- Now you get the marginal risk amount the portfolio in question contributes to the total portfolio
  - e.g. The discrete marginal contribution of portfolio X is 37.0 - 25.9 = 11.1
- The marginal contribution is often scaled to get the marginal contributions that add up to the total portfolio risk
- Discrete marginal contributions are an approximation to the continuous marginal contributions approach

Portfolio	VaR of Total Portfolio Excluding This One	Discrete Marginal Contribution	Scaled Marginal Contribution
X	25.9	11.1	13.6 =(37/30) x 11.1
Y	31.4	5.6	6.9 =(37/30) x 5.6
Z	23.6	13.4	16.5 =(37/30) x 13.4
XYZ		30.0	37

<u>Continous Marginal Contributions</u>

- Known as the Euler Method
- Calculate the derivative of the total portfolio risk with respect to each individual portfolio's risk
- Multiply the derivatives by the individual portfolio's risk measures
- This gives you the continuous marginal contribution of that portfolio
- Advantage: The continuous marginal contributions of the portfolios, add up to the total capital of the portfolio
- Disadvantage: There can be negative contributions where negative correlations exist between the individual portfolios

Assume the capital required for X is 15.9, for Y is 8.8, and for Z is 22.7. The total capital required for XYZ with diversification is 37.

Assume that for a 1% increase in the VaR of portfolio X, portfolio XYZ's capital requirement will increase 0.81%.

Portfolio	Change VAR / Change Portfolio VAR	Continuous Marginal Contribution
X	0.81	12.9 = (0.81 x 15.9)
Y	0.71	6.2 = (0.71 x 8.8)
Z	0.79	17.9 = (0.79 x 22.7)
XYZ		37 = (12.9 + 6.2 + 17.9)

Myers-Read allocation method:

- Its advantage over the other marginal methods is that the marginal increments add up to the total capital
- Shareholders of a firm hold a put option on the insurer's losses that exceed the capital the insurer has available
  - The shareholders can put these losses to the policyholders as the shareholders are only liable for losses while the insurer is solvent
- Myers-Read usually assumes a normal or lognormal total loss distribution
- The value of the default put option can then be determined via the Black-Scholes option pricing formula
- Increasing risk exposure of any portfolio leads to a larger value of the default put option on that portfolio
- Adding capital to the portfolio decreases the value of the default put option
- Myers-Read calculates the cost of the last unit of exposure added to a portfolio in terms of the addition to the capital required to support that unit of exposure
- The additional capital required in a portfolio when an additional unit of risk is added is calculated such that the value of the default put option as a percentage of expected losses remains constant
- The additional costs of the last unit added is applied to all the units in the portfolio to arrive at the total risk capital for the portfolio

Game theory

Game theory:

- Widely used approach to decision making in conflict situations
- In risk allocation, the conflict is how to share the diversification benefit between each sub portfolio
- Each sub-portfolio benefits from being part of a larger diversified portfolio, but also gives up some diversification benefit

Shapley Value:

- Introduced by Lloyd Shapley as a stable solution to coalition games
- There can be any number of players
- The number of players needs to be a whole number — seen as a limitation
- The Shapley Value is based on the average first in, last in and all intermediate ins.
- The second in calculations are calculated as if the portfolio was the second entrant into each possible combination of portfolios
- No scaling is needed
- The allocations to each portfolio naturally adds up to the total

Instructor's Note: Work through the example on your own and we'll go through it in the online seminar.

Drawbacks of the Shapley Value:

1. The calculation is computation intensive since the number of calculations increase exponentially for more portfolios
2. There is the issue of needing a whole number of players. This is not a desirable property of an allocation method.

Aumann-Shapley Value:

- Developed by Aumann and Shapley which allows for fractional players
- It represents the rate of increase in risk
  - How much additional risk a portfolio adds for a small increase in size?
- The risk is calculated based on the risk measure adopted.
- The Myers-Read approach is considered a special case of Aumann-Shapley, where the risk measure used is the default put option
- Aumann-Shapley values can be calculated using simulation techniques

Other Approaches

- Co-measures
  - Developed as a way of allocating capital in an additive manner that is still consistent with the risk measure used to define total capital
  - e.g. The risk measure can be TVaR, then the co-TVaR for a portfolio would be the average of the losses that the portfolio contributes to the total TVaR. The co-TVaR's add up to the total TVaR
- Allocate capital so it equalizes relative risk
  - Each portfolio would be allocated risk such that, when viewed in isolation, each portfolio has the same ratio of risk to expected losses

