Option Pricing Basics - Outline

1. A.2.1 Put-Call Parity
    - Put-Call Parity Formula
    - Synthetically Creating Assets
    - Exploiting Arbitrage

For a European call and put with the same K and T, **put-call parity** (PCP) describes the no-arbitrage relationship between the options' premiums:

$$C(S, K, T) - P(S, K, T) = F^P_{0,T}(S) - F^P_{0,T}(K)$$

E.g., if S is a non-dividend paying stock and K is cash, then

$$C(S, K, T) - P(S, K, T) = S_0 - Ke^{-rT}$$

Put-Call Parity Proof

Consider a portfolio that buys a non-dividend paying share of stock, buys a European put option on the stock and borrows $\$Ke^{-rT}$

		Payoff at T	
	CF Today	$S_T < K$	$S_T > K$
Buy stock	$-S_0$	S_T	S_T
Buy put	$-P$	$K - S_T$	0
Borrow $\$Ke^{-rT}$	$+K\,e^{-rT}$	$-K$	$-K$
	$-S_0 - P + K\,e^{-rT}$	0	$S_T - K$

Note that this portfolio has the *exact* same payoff as $C(S, K, T)$

Put-Call Parity Proof *(continued)*

By the law of one price, two portfolios with the exact same payoff must have the same price. Thus:

$$\text{Cashflow to buy call} = \text{CF to buy portfolio}$$

$$-C = -S_0 - P + K\,e^{-rT}$$

$$C - P = S_0 - K\,e^{-rT}$$

We can use the PCP equation if we wish to synthetically create (i.e., replicate) one of the assets found in the PCP equation:

1. Rearrange the equation to isolate the variable representing the CF to enter the position you wish to create
2. Opposite side of equation describes CFs of transactions that will synthetically create the desired position

$$C - P = S_0 - Ke^{-rT}$$

$$\underbrace{-C}_{\text{long call}} = \underbrace{-P}_{\text{buy put}} - \underbrace{S_0}_{\text{buy stock}} + \underbrace{Ke^{-rT}}_{\text{borrow}}$$

Synthetically Creating Assets: Example

Example

How would you synthetically create a long position in a European put option on a stock that pays dividends continuously at annual yield δ using shares of the stock, a European call option and borrowing or lending?

1. The relevant put-call parity equation is:

$$C - P = Se^{-\delta T} - Ke^{-rT}$$

2. Rearranging to isolate the cashflow from purchasing a put, we get:

$$-P = Se^{-\delta T} - Ke^{-rT} - C$$

Example *(continued)*

$$-P = Se^{-\delta T} - Ke^{-rT} - C$$

Thus, we see that we can replicate the put option with the following three transactions:

- Sell $e^{-\delta T}$ shares of stock
- Buy the call option
- Lend Ke^{-rT} dollars at the risk-free rate
  - Note that this is the same as investing $\$Ke^{-rT}$ or purchasing a risk-free zero-coupon bond with a face value of $\$K$

When a no-arbitrage parity condition is violated, arbitrage is available. A useful trick to know *how* to exploit arbitrage opportunities dealing with asset prices is to do the following:

1. Write down the observed inequality
2. Move everything to the "greater than" side of the inequality
3. The resulting symbols represent the cash flows from the transactions that exploit the arbitrage

Exploiting Arbitrage: Example

Consider a European call and put option on a non-dividend paying stock. You observe that the call option's actual premium is higher than the price implied by put-call parity. How would you exploit the arbitrage?

1. You observed: $C > S_0 - Ke^{-rT} + P$
2. Moving everything to the "greater than" side gives:

$$C - S_0 + Ke^{-rT} - P > 0$$

3. You can exploit the arbitrage by doing the following:
   - Sell the call
   - Buy the stock
   - Buy the put
   - Borrow Ke^{-rT}