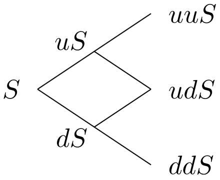
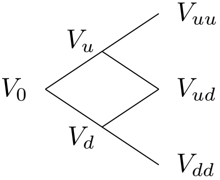
Black-Scholes Option Greeks - Outline

1. C.2.2 Greeks in the Binomial Model
   - Estimating Delta
   - Estimating Gamma
   - Estimating Theta

Estimating Greeks in a Binomial Setting

Although we introduced the Greeks in a B-S framework, we can also estimate Δ, Γ and θ in a binomial framework



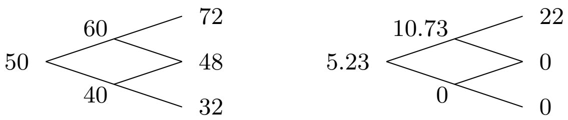


We have previously seen how to calculate Δ in a binomial model. At any node j:

$$\Delta_j = e^{-\delta h}\left(\frac{V_{ju} - V_{jd}}{uS_j - dS_j}\right)$$

Example

Consider a two-period binomial model with h=1 year. The values throughout the tree of the underlying stock and a call option are shown below at left and right, respectively.



If $\delta = 5\%$, estimate Δ at nodes u and d.

1. $\Delta_u = e^{-.05(1)}\left(\frac{22-0}{72-48}\right) = 0.8720$
2. $\Delta_d = e^{-.05(1)}\left(\frac{0-0}{48-32}\right) = 0$

Since Γ is the second derivative of V with respect to S, we could also define it as:

$$\Gamma = \frac{\partial \Delta}{\partial S}$$

Using this interpretation, we can estimate Γ at time 0 in a binomial model as:

$$\Gamma_0 = \frac{\Delta_u - \Delta_d}{uS - dS}$$

Estimating Gamma: Example

Example

Consider again the previous example. Estimate the option's time-0 Γ.

$$\Gamma_0 = \frac{\Delta_u - \Delta_d}{uS - dS} = \frac{0.8720 - 0}{60 - 40} = 0.0436$$

Theta in the Binomial Model

Recall that θ is the change in V with respect to a move forward in time. To best isolate the effect of time, we estimate θ in a binomial model using the change from V_0 to V_{ud}

The reasoning is that udS is expected to be reasonably close to S, minimizing the impact of the change in S on the change in V

Define $\epsilon = udS - S$. To estimate θ, we start with the $\Delta - \Gamma - \theta$ approximation:

$$V_{ud} - V_0 = \Delta_0 \epsilon + \frac{1}{2}\Gamma_0 \epsilon^2 + 2h\theta_0$$

Solving for θ we get:

$$\theta_0 = \frac{V_{ud} - V_0 - \Delta_0 \epsilon - \frac{1}{2}\Gamma_0 \epsilon^2}{2h}$$

Note that when $\epsilon = 0$, this simplifies to $\theta_0 = \frac{V_{ud} - V_0}{2h}$

Example

Consider once more the previous example. Estimate the option's time-0 θ.

1. $\epsilon = udS - S = 48 - 50 = -2$
2. $\Delta_0 = e^{-\delta h}\left(\frac{C_u - C_d}{uS - dS}\right) = e^{-.05(1)}\left(\frac{10.73-0}{60-40}\right) = 0.5103$
3. $\theta_0 = \frac{C_{ud} - C_0 - \Delta_0 \epsilon - \frac{1}{2}\Gamma_0 \epsilon^2}{2h}$

$$= \frac{0 - 5.23 - 0.5103(-2) - \frac{1}{2}(0.0436)(-2)^2}{2(1)} = -2.1483$$

Note that since h was entered in years, this is an annual θ (i.e., the change in option value with respect to a one year change in time). The daily θ is given by $(1/365)(-2.1483)$.