Credible Claims Reserve: Benktander, Neuhaus and Mack
W. Hürlimann

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Calculate unpaid claim estimates using credibility models. Range of weight: 10-14 percent	a. Application of credibility b. Mechanics of the methods (including loss ratio based payout factors) c. Strengths and weaknesses d. Testing results for reasonableness

READINGS
• Brosius • Hurlimann • Mack (2000)

Synopsis
The author introduces a new way to estimate the reporting pattern p_k. It uses **loss ratios** by age, rather than development of losses. With this payment pattern, the author explores methods to estimate **unpaid losses**.

The Chainladder (CL) method uses actual losses to estimate the unpaid losses.

The Bornhuetter-Ferguson (BF) method uses an *a priori* to estimate the unpaid losses.

The Benktander (GB) method gives credibility $Z = p_k$ to the Chainladder method.

The author introduces two other methods:

Neuhaus (WN) with $Z = p_k \cdot Expected\ Loss\ Ratio$

Optimal Credibility with $Z = \frac{p}{p+\sqrt{p}}$

This latter method minimizes the Mean Squared Error of the Reserve Estimate.

A note added in the 2016 syllabus: "Candidates are not responsible for mathematical proofs". This makes explicit that you don't need to be able to do the proofs.

1. Introduction
Similar to the Cape Cod method, this method uses an exposure base (premium), and all the losses in the triangle to determine an *a priori* loss ratio for the triangle.

Notation
The paper assumes we have an $n \times n$ triangle, and that losses are fully developed at age n.

S_{ik} = Incremental Paid Losses
C_{ik} = Cumulative Paid Losses for accident year i, at age k
U_i = Ultimate Losses for period i
V_i = exposure base for period i (eg. premium)

m_k = expected loss ratio at development age k (column k)
$\widehat{m}_k$ = estimate of m_k (*this is my notation, not the author's*)

In this note we refer to Accident Years for simplicity, but they could be Report Years, Underwriting or Policy Years. They could also be periods that are more or less than a year.

2. Collective and Individual Loss Ratio Claims Reserves

The focus on this paper is on the expected **loss ratio** for each **age**. We call these m_k, and we estimate by summing the losses down each column, and dividing by the associated premium.

Incremental Paid Losses

S_{ik}	1	2	3	Premium (V_i)
2001	102	29	17	300
2002	114	35		350
2003	118			400
$\widehat{m}_k$	31.8%	9.8%	5.7%	

$$\widehat{m}_1 = \frac{102 + 114 + 118}{300 + 350 + 400} = \frac{334}{1{,}050} = 31.8\%$$

$$\widehat{m}_2 = \frac{29 + 35}{300 + 350} = \frac{64}{650} = 9.8\%$$

$$\widehat{m}_3 = \frac{17}{300} = 5.7\%$$

We can sum these $\widehat{m}_k$'s to estimate the expected loss ratio (ELR). This ELR is used for the entire triangle, and is used as the *a priori* for the BF method.

$$ELR = \sum_{k=1}^{n} \widehat{m}_k$$

$$ELR = 31.8\% + 9.8\% + 5.7\% = \mathbf{47.3}\%$$

The author uses the notation p_i to represent the % paid to date for accident year i.
I'm going to deviate from this notation.
I find it much more intuitive to focus on $\boldsymbol{p_k}$ the % paid to date at **age $\boldsymbol{k}$**. This is the same notation used in Mack (2000).

We can use these m_k's to estimate the % of losses emerged: $\boldsymbol{p_k}$ at age k. The author calls these **loss ratio payout factor** or **loss ratio lag-factor**.

$$p_k = \frac{1}{ELR} \cdot \sum_{j=1}^{k} m_j$$

Using the above triangle, we get:

$$p_1 = \frac{31.8\%}{47.3\%} = 0.672$$

$$p_2 = \frac{31.8\% + 9.8\%}{47.3\%} = 0.879$$

$$p_3 = 1.000$$

The complement is referred to as the **loss ratio reserve factor**:
$\boldsymbol{q_k} = 1 - p_k$

Individual Loss Ratio Claims Reserve

Now, we use what is effectively the Chain Ladder method. We estimate ultimate losses by taking the paid to date, and dividing by p_k.

$$\boldsymbol{U_i^{ind} = \frac{C_{ik}}{p_k}}$$

Eg.

$$U_{2002}^{ind} = \frac{114 + 35}{0.879} = 169.5$$

$$U_{2003}^{ind} = \frac{118}{0.672} = 175.6$$

We can also calculate the reserve:

$$\boldsymbol{R_i^{ind}} = \frac{C_{ik}}{p_k} \cdot q_k$$

We can also write:

$$\boldsymbol{R_i^{ind} = \frac{C_{ik}}{p_k} - C_{ik}}$$

I prefer the second form since subtracting paid to date always gets us from Ultimate to the reserve. Multiplying by q_k only works with the CL method.
Eg.

$$R_{2002}^{ind} = \frac{114 + 35}{0.879} - (114 + 35) = 20.5$$

So far all these formulas look just like the ones in Mack(2000). The only difference is that p_k and q_k are calculated using the loss ratio method instead of Age to Age factors

Collective Loss Ratio Claims Reserve

This is the equivalent to the BF method. We first need *a priori* losses for each year (U_i^{BC}) (BC stands for Burning Cost). We estimate U_i^{BC} by multiplying the premium for the year (V_i), and the expected loss ratio for the triangle (ELR).

$$U_i^{BC} = V_i \cdot ELR$$

Then, the reserve is simply the *a priori* times the % unpaid:

$$R_i^{coll} = q_k \cdot U_i^{BC}$$

I think it's easier to remember:

$$\boldsymbol{R_i^{coll} = q_k \cdot (V_i \cdot ELR)}$$

Which is similar to what we saw in Mack(2000)

The estimate of ultimate losses is simply the paid to date plus the reserve:

$$U_i^{coll} = C_{ik} + R_i^{coll}$$

Example

a)

Given the following triangle of paid losses, calculate the loss ratio payout factors.

	1	2	3	4	Premium
2013	217	104	58	29	500
2014	251	134	57		580
2015	215	63			600
2016	240				640

Solution:

a)

	1	2	3	4	Premium
2013	217	104	58	29	500
2014	251	134	57		580
2015	215	63			600
2016	240				640

m_k	0.398	0.179	0.106	0.058	0.741
p_k	0.537	0.779	0.922	1.000	

$$m_1 = \frac{217 + 251 + 215 + 240}{500 + 580 + 600 + 640} = \frac{923}{2{,}320} = 0.398$$

$$m_2 = \frac{104 + 134 + 63}{500 + 580 + 600} = \frac{301}{1{,}680} = 0.179$$

$$m_3 = \frac{58 + 57}{500 + 580} = \frac{115}{1{,}080} = 0.106$$

$$m_4 = \frac{29}{500} = 0.058$$

$$ELR = 0.398 + 0.179 + 0.106 + 0.058 = 0.741$$

$$p_1 = \frac{0.398}{0.741} = 0.537$$

$$p_2 = \frac{0.398 + 0.179}{0.741} = 0.779$$

$$p_3 = \frac{0.398 + 0.179 + 0.106}{0.741} = 0.922$$

$$p_4 = 1.000$$

Example Continued

b) Calculate the Individual Loss Ratio Claims Reserve for each accident year

c) Calculate the Collective Loss Ratio Claims Reserve for each accident year

Solution
b)

	Paid	p	U^{ind}	R^{ind}
2013	408	1.000	408.0	**-**
2014	442	0.922	479.4	**37.4**
2015	278	0.779	356.9	**78.9**
2016	240	0.537	446.9	**206.9**
				323.2

2015:

$$C_{ik} = 215 + 63 = 278$$

$$\frac{C_{ik}}{p_k} = \frac{278}{0.779} = 356.9; \quad 356.9 - 278 = 78.9$$

c)

$ELR = 74.1\%$

	Premium	$Prem \cdot ELR$	p	q	R^{coll}
2013	500	370.5	1.000	-	**-**
2014	580	429.8	0.922	0.078	**33.5**
2015	600	444.6	0.779	0.221	**98.3**
2016	640	474.2	0.537	0.463	**219.6**
					351.4

2015:

$$Prem \cdot ELR = 600 \cdot 0.741 = 444.6$$

$$R^{coll} = q \cdot (V \cdot ELR) = 0.221 \cdot 444.6 = 98.3$$

Author states one of the benefits of the collective method is that two actuaries, using the same premium, will come up with the same answer. In contrast, the BF method, is dependent on the *a priori* loss ratio that the actuary selects.

3. Credible Loss Ratio Claims Reserve

The Individual Claims reserve gives full credibility to that accident year's losses. The Collective Claims Reserves gives zero credibility. So, we will credibility weight them with a Z_i that varies by accident year.

$$\boldsymbol{R_i^c = Z_i \cdot R_i^{ind} + (1 - Z_i) \cdot R_i^{coll}}$$

Z_i	Method	Alternate Name
1	Individual Loss Ratio	Chainladder
0	Collective Loss Ratio	Bornhuetter-Ferguson
p_k	Benktander (GB)	
$p_k \cdot ELR$	Neuhaus (WN)	
$\dfrac{p_k}{p_k + \sqrt{p_k}}$	Optimal Credibility	

Neuhaus assigns credibility equal to the expected loss ratio to date. So, if the expected loss ratio is 60%, and losses are expected to be 40% reported, than the credibility is $24\% = 60\% \cdot 40\%$.
Thus, as the year develops, the expected reported loss ratio increases and thus so does the credibility.
One consequence of using loss ratio is that changing the exposure base will change the result.
Neuhaus assigns low credibility to lines with low loss ratios.

The author states that it is remarkable that in numerical examples, both Benktander and Neuhaus are close to an *optimal credible loss ratio claims reserve*.
Let's calculate R_{2003} using each method:

We have: $C_{2003,1} = 118;\ p_1 = 0.672;$

$$q_1 = 0.328;\ ELR = 47.3\%;\ V_{2003} = 400$$

$$R_{2003}^{ind} = \frac{C_{2003,1}}{p_1} - C_{2003,1} = \frac{118}{0.672} - 118 = \mathbf{57.6}$$

$$R_{2003}^{coll} = q_1 \cdot (V_{2003} \cdot ELR) = 0.328 \cdot (400 \cdot 47.3\%) = \mathbf{62.1}$$

Benktander: $Z_{2003}^{GB} = p = 0.672$

$$R_{2003}^{GB} = 0.672 \cdot 57.6 + (1 - 0.672) \cdot 62.1 = \mathbf{59.1}$$

Neuhaus: $Z_{2003}^{WN} = 0.672 \cdot 47.3\% = 0.318$

$$R_{2003}^{WN} = 0.318 \cdot 57.6 + (1 - 0.318) \cdot 62.1 = \mathbf{60.7}$$

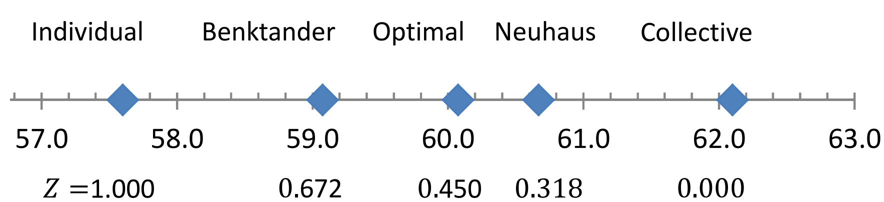
Optimal: $Z_{2003}^{c} = \frac{0.672}{0.672+\sqrt{0.672}} = 0.450$

$$R_{2003}^{c} = 0.450 \cdot 57.6 + (1 - 0.450) \cdot 62.1 = \mathbf{60.1}$$

Here is the triangle again, for reference

Incremental Paid Losses

S_{ik}	1	2	3	Premium (V_i)
2001	102	29	17	300
2002	114	35		350
2003	118			400
$\hat{m}_k$	31.8%	9.8%	5.7%	
p_k	0.672	0.879	1.000	
q_k	0.328	0.121	-	



Theorem 3.1
As is noted in the Mack(2000) paper, the Benktander method ($R^{GB} = q_k U^{BF}$) takes the Ultimate estimate from the BF method, and then uses that as the *a priori* for the credibility weighing. This effectively gives weight q_k^2 to the *a priori* U^0, and weight $1 - q_k^2$ to the CL method.

$$U^{GB} = (1 - q^2) \cdot U^{CL} + q^2 \cdot U^0$$

Proof:

$$U^{BF} = C + qU^0$$

$$R^{GB} = q \cdot U^{BF} = q \cdot C + q^2 U^0$$

$$U^{GB} = C + R^{GB} = C + q \cdot C + q^2 U^0$$

$$U^{GB} = C \cdot (1 + q) + q^2 U^0$$

$$U^{GB} = \frac{C(1 + q) \cdot (1 - q)}{1 - q} + q^{2U^0}$$

$$U^{GB} = \frac{C}{p} \cdot (1 - q^2) + q^2 U_0$$

$$\boldsymbol{U^{GB} = (1 - q^2)U^{CL} + q^2 U^0}$$

One could iterate this many times (say m times). Then, the weight given to the *a priori* is q_k^m. As $m \to \infty$, then the weight applied to U^0 approaches zero, and we give full credibility to the CL method.

4. The Optimal Credibility Weights and the Mean Squared Error

This section gets theoretical. Here we will summarize the results of the theorems. In an appendix to the manual, we will list out the assumptions in more detail. In this theoretical section, due to the large number of formulas, we revert to the author's definition of p_i – the expected percent paid to date for Accident Year i.

We would like to find a credibility Z_i that minimizes the $\boldsymbol{MSE(R_i^c) = E[(R_i^c - R_i)^2]}$

Theorem 4.1

By making an assumption that U_i^{BC} is independent from C_i and R_i, the author shows that the credibility Z_i^* that minimizes $MSE(R_i^c)$ is:

$$Z_i^* = \frac{p_i}{q_i}\frac{Cov(C_i, R_i) + p_i q_i \cdot Var(U_i^{BC})}{Var(C_i) + p_i^2 \cdot Var(U_i^{BC})}$$

Some comments we can make about this Z_i

- Since $\frac{p_i}{q_i}$ increases as losses emerge, then Z_i^* increases as losses emerge. This is appropriate.
- The $Cov(C_i, R_i)$ term measures the covariance for the accident year of losses paid to date, and the unpaid losses
  The larger the covariance, the large Z_i. This makes sense, since a large covariance implies that C_i is predictive of R_i
- If the Variance of losses paid to date, $Var(C_i)$, is high, then the fractional term is small, and we have a low credibility. Again, this is appropriate, if C_i is volatile, we don't want to rely too much on CL to estimate the reserves
- Finally, if the $Var(U_i^{BC})$ term is large, then $Z_i^* \sim \frac{p_i}{q_i}\frac{p_i q_i}{p_i^2} \sim 1$. This is reasonable, since U_i^{BC} is the complement. If the variability of the complement is large, you trust the CL more.

For the following theorems, we make the following assumptions (4.4):

$$E\left[\frac{C_i}{U_i}\middle|U_i\right] = p_i$$

$$Var\left[\frac{C_i}{U_i}\middle|U_i\right] = p_i q_i \beta_i^2(U_i)$$

We define: $\alpha_i^2(U_i) = U_i^2 \beta_i^2(U_i)$

Theorem 4.2 gives us the following:

$$Z_i^* = \frac{p_i}{p_i + t_i}$$

Theorem 4.3 builds off the other theorems, and allows us to calculate the mean squared error for the reserve estimates: optimal, individual, and collective:

$$mse(R_i^c) = E[\alpha^2(U_i)] \cdot \left[\frac{Z_i^2}{p_i} + \frac{1}{q_i} + \frac{(1-Z_i)^2}{t_i}\right] \cdot q_i^2$$

By plugging in $Z = 1$, and $Z = 0$, we also get the MSE for the individual and collective methods:

$$mse\left(R_i^{ind}\right) = E[\alpha^2(U_i)] \cdot \frac{q_i}{p_i}$$

$$mse\left(R_i^{coll}\right) = E[\alpha^2(U_i)] \cdot q_i \cdot \left(1 + \frac{q_i}{t_i}\right)$$

5. A Pragmatic Estimation Method

We expect that:

$$\widehat{Var}(U_i) > \widehat{Var}(U_i^{BC})$$

That is the variance of Ultimate losses is greater than the burning cost estimate.
The author makes the assumption that

$$\widehat{Var}(U_i) = f_i \cdot \widehat{Var}(U_i^{BC})$$

Where $f_i > 1$

6. The Optimal Credible Claims Reserve with Minimum Variance

If we make the assumption that $\beta_i(U_i) = \beta_i$; is a constant, and that $f_i = 1$, then we get:

$$t_k \sim \sqrt{p_i}$$

This selection $f_i = 1$ gives a lower credibility to the Individual method than other selections for f_i.

This finally, gives us:

$$Z_i^* = \frac{p_i}{p_i + \sqrt{p_i}}$$

Using the notation that p_k is the expected percent paid at age k, we have the main formula from the paper:

$$\boldsymbol{Z_i^* = \frac{p_k}{p_k + \sqrt{p_k}}}$$

Remark 6.1
See errata at the end of this manual for a comment if you are reading the source.

Up to now we've been using p_k as the **% Paid**, based on our estimates of m_k, the column loss ratios. The authors suggest by defining p_k^{CL} as the % Paid implicit from the loss development factors, these same credibility methods can be used. One would simply replace p_k with p_k^{CL} in each calculation.

Using the triangle from above, we'd get:

Incremental Paid Losses

S_{ik}	1	2	3	Premium (V_i)
2001	102	29	17	300
2002	114	35		350
2003	118			400
Incremental LDF	1.296	1.130		
Cumulative LDF	1.464	1.130	1.000	
p_k^{CL}	0.683	0.885	1.000	
p_k	0.672	0.879	1.000	

To estimate the unpaid losses for **Accident Year 2003**, we'd do the same formulas as above, except with $p = 0.683$, instead of $p = 0.672$. The other change we have to make is to the ELR for the Collective Reserve R^{coll}.

In the following formulas, we will drop the subscripts.

$$R^{ind} = \frac{C}{p} - C$$

$$R^{ind} = \frac{118}{0.683} - 118 = \mathbf{54.8}$$

Cape Cod Method
We estimate the ELR using Cape Cod, and this gives us the Collective Reserve, and then we credibility weight.

$$\boldsymbol{ELR} = \frac{(102 + 29 + 17) + (114 + 35) + (118)}{300 \times 1.000 + 350 \times 0.885 + 400 \times 0.683} = \frac{415.0}{883.0} = 47.0\%$$

$$\boldsymbol{R^{Coll}} = \boldsymbol{q} \cdot (\boldsymbol{V} \times \boldsymbol{ELR})$$

$$\boldsymbol{R^{Coll}} = (1 - 0.683)(400 \times 47.0\%) = \mathbf{59.6}$$

Benktander

For Benktander, we use $Z = p$

$$Z = p = 0.683$$

$$R^{GB} = Z \cdot R^{ind} + (1 - Z) \cdot R^{coll}$$

$$\boldsymbol{R^{GB}} = 0.683 \cdot 54.8 + (1 - 0.683) \cdot 59.6 = \mathbf{56.3}$$

Optimal Cape Cod Method
Here we use the optimal credibility with the Cape Cod ELR

$$Z = \frac{p}{p + \sqrt{p}} = \frac{0.683}{0.683 + \sqrt{0.683}} = 0.452$$

$$R^{c} = 0.452 \times 54.8 + (1 - 0.452) \times 59.6 = \mathbf{57.4}$$

7. Numerical Examples

The excel file [Hurlimann.xls] does the following:

- Shows calculation of m_k
- Calculation of R_i^{ind} and R_i^{coll}, as well as $R_i^{GB}, R_i^{WN}, R_i^C$
- Also compares the p_k from Hurlimann, to the p_k from using Age to Age factors

We are given the following incremental paid losses, and we will use various methods to estimate the Unpaid Losses.

Incremental Paid Losses

Period (i)	1	2	3	4	5	6	Cumulative Paid	Earned Premium
2001	4,370	1,923	3,999	2,168	1,200	647	14,307	13,085
2002	2,701	2,590	1,871	1,783	393		9,338	14,258
2003	4,483	2,246	3,345	1,068			11,142	16,114
2004	3,254	2,550	2,547				8,351	15,142
2005	8,010	4,108					12,118	16,905
2006	5,582						5,582	20,224

We first find the column loss ratios

$\boldsymbol{m_k}$	29.67%	17.77%	20.07%	**11.55%**	5.83%	4.94%	89.83%

$$m_4 = \frac{2{,}168 + 1{,}783 + 1{,}068}{13{,}085 + 14{,}258 + 16{,}114} = \frac{5{,}019}{43{,}457} = 11.55\%$$

We can then calculate the payout factors:

Age k

Period(i)	1	2	3	4	5	6
p_k	0.330	0.528	0.752	0.880	0.945	1.000
q_k	0.670	0.472	0.248	0.120	0.055	-

$$p_4 = \frac{29.67 + 17.77 + 20.07 + 11.55}{89.83} = 0.880$$

Here, we calculate the credibility for each AY

AY	Age k	p_k	Credibility Benktander	Neuhaus	Optimal
2001	6	1.000	1.000	0.898	0.500
2002	5	0.945	0.945	0.849	0.493
2003	4	0.880	0.880	0.791	0.484
2004	**3**	**0.752**	**0.752**	**0.676**	**0.464**
2005	2	0.528	0.528	0.474	0.421
2006	1	0.330	0.330	0.296	0.365

AY 2004:
Differences from the text are due to rounding error

$$Z^{GB} = p_3 = 0.752$$

$$Z^{WN} = p_3 \cdot ELR = 0.752 \cdot 89.83\% = 0.676$$

$$Z^c = \frac{p_3}{p_3 + \sqrt{p_3}} = 0.464$$

$$\boldsymbol{R^{ind}} = \frac{C}{p} - C = \frac{8{,}351}{0.752} - 8{,}351 = \mathbf{2{,}754}$$

$$\boldsymbol{R^{coll}} = q \cdot (V \cdot ELR) = (1 - 0.752) \cdot (15{,}142 \cdot 89.83\%) = \mathbf{3{,}373}$$

		In Text:
R^{GB}	$= 0.752 \cdot 2{,}754 + (1 - 0.752) \cdot 3{,}373 = \mathbf{2,908}$	2,915
R^{WN}	$= 0.676 \cdot 2{,}754 + (1 - 0.676) \cdot 3{,}373 = \mathbf{2,955}$	2,962
R^{c}	$= 0.464 \cdot 2{,}754 + (1 - 0.464) \cdot 3{,}373 = \mathbf{3,086}$	3,092

This table has the reserves for all the periods:

Table 7.3

Period	Collective	Individual	Neuhaus	Benktander	Optimal
2	705	544	568	553	626
3	1,736	1,518	1,564	1,544	1,630
4	**3,380**	**2,761**	**2,962**	**2,915**	**3,092**
5	7,166	10,829	8,904	9,101	8,708
6	12,167	11,320	11,916	11,887	11,858
All	25,154	26,972	25,913	25,999	25,914

Note that Table 7.4 in the text (not shown here) has errors – they are discussed at the end of the manual.

This table shows the Mean Squared error around each estimate, in proportion to the MSE of the Optimal estimate

Period	Collective	Individual	Neuhaus	Benktander	Optimal
2	1.03	1.03	1.01	1.02	1.00
3	1.06	1.07	1.00	1.04	1.00
4	1.12	1.15	1.00	1.04	1.00
5	1.20	1.38	1.12	1.01	1.00
6	1.24	1.74	1.41	1.00	1.00

This table is color coded for values below 1.05, 1.10, and 1.20.
Notice the significant improvement in MSE of Neuhaus and Benktander against either the Individual (equivalent to Chainladder) or Collective (equivalent to BF or Cape Cod). The Optimal Credibility Method is a further improvement.

2nd Example

We have this data set of modified actual losses.

Incremental Paid Losses

Period (i)	1	2	3	4	5	6	Cumulative Paid	Earned Premium
1	3,789	2,861	507	152	65	24	7,398	8,000
2	3,583	2,687	1,250	536	880		8,936	9,000
3	4,222	3,166	2,249	208			9,845	10,000
4	4,074	2,950	1,163				8,187	10,000
5	1,228	3,907					5,135	10,000
6	6,840						6,840	12,000

We calculate the column loss ratios:

m_k	40.2%	33.1%	14.0%	3.3%	5.6%	0.3%	96.5%
p_k	0.417	0.760	0.905	0.939	0.997	1.000	

The estimated reserves are:

Period	Collective	Individual	Neuhaus	Benktander	Optimal
2	27	28	28	28	28
3	586	637	632	634	611
4	918	861	868	866	890
5	2,315	1,620	1,805	1,787	1,991
6	6,754	9,569	7,886	7,927	7,858
All	10,600	12,715	11,219	11,242	11,378

Notice the large differences in the reserve estimate for the two most recent accident years.

The table has the Ratio of MSE to the MSE of the Optimal Credibility method.

Period	Collective	Individual	Neuhaus	Benktander	Optimal
2	1.00	1.00	1.00	1.00	1.00
3	1.03	1.03	1.02	1.02	1.00
4	1.05	1.05	1.03	1.03	1.00
5	1.11	1.15	1.04	1.04	1.00
6	1.23	1.55	1.00	1.00	1.00

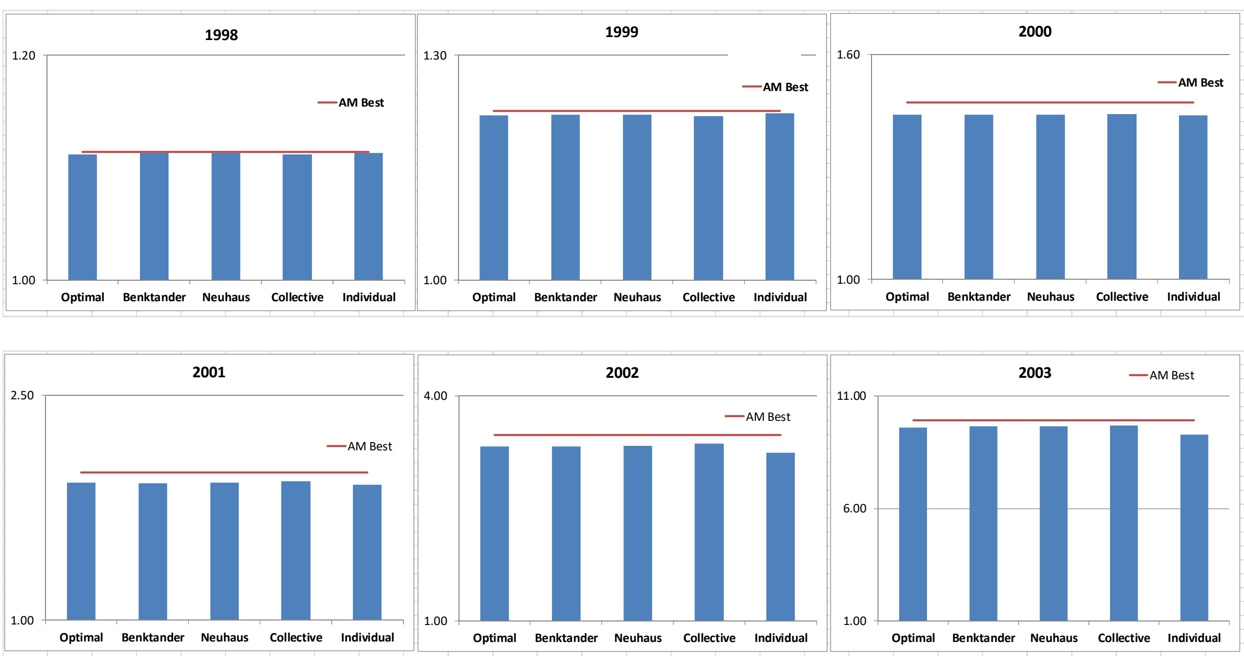
Again, Neuhaus and Benktander have lower MSE than the Individual and Collective Reserve methods. Optimal is slightly better.

AM Best Example

In this example, the author takes a Loss Triangle for General Liability Claims Made policies from the 2004 AM Best tables. In addition to the triangle, AM Best provides a set of selected LDFs. Using the triangle of losses from AM Best, we estimate ultimate losses using each method. In this example, rather than comparing the Ultimate losses; we calculate an implicit LDF by dividing the Ultimate losses by the losses paid to date – for each method and compare those to the LDFs selected by AM Best.

Year	**AM Best**	**Optimal**	Benktander	Neuhaus	Collective	Individual
1997	1.066	1.062	1.062	1.062	1.061	1.062
1998	1.114	1.112	1.113	1.113	1.112	1.113
1999	1.226	1.220	1.221	1.221	1.219	1.222
2000	1.471	1.439	1.439	1.439	1.441	1.438
2001	1.986	1.917	1.914	1.915	1.927	1.903
2002	3.475	3.322	3.328	3.331	3.366	3.245
2003	9.903	9.595	9.652	9.655	9.696	9.285

In the graphs below, we show the LDF from each of the 5 methods as the blue bars, and the selected LDF from AM Best as the red line.
For 1997 – 1999, the AM Best result is consistent with the other results (1997 is not graphed). For 2000 the AM Best result is somewhat higher.
For 2001 – 2003 it is clear that the AM Best factors systematically overstate the optimal LDF and the nearly optimal Benktander and Neuhaus factors.



Similar results hold for other insurance categories provided by AM Best.

Appendix

In assumption (4.4), we write the variance as:

$$Var\left[\frac{C_i}{U_i}\middle|U_i\right] = p_i q_i \beta_i^2(U_i)$$

Having q_i in the formula assures us that when the year is fully developed, the variance is 0.
Having p_i also assures us that the variance is small when the expected reported is small.
The function $\beta_i^2(U_i)$ is undefined.

Theorem 4.2
By making the assumption (4.4), we get the following results:

(4.5)

$$Z_i^* = \frac{p_i}{p_i + t_i}$$

(4.6)

$$t_i = \frac{E[\alpha_i^2(U_i)]}{Var[U_i^{BC}] + Var[U_i] - E[\alpha_i^2(U_i)]}$$

5. A Pragmatic Estimation Method
Theorem 6.1
If we assume that $\beta_i^2(U_i) = \beta_i^2$; a constant, then we can further simplify the formula for t_i

$$t_i = \frac{1}{2} \cdot \left[f_i - 1 + \sqrt{(f_i + 1) \cdot (f_i - 1 + 2p_i)}\right]$$

If we further assume that $f_i = 1$, then we get:

$$t = \sqrt{p}$$

Summary of Paper

- $m_k =$ loss ratio in a column: sum of losses / sum of premium
- Calculate p_k, the % paid to date, by using the loss ratios from m_k
- R^{ind} and R^{coll} are effectively the Chainladder and Bornhuetter-Ferguson methods
- $R^{ind} = \frac{C_{ik}}{p_k} - C_{ik}$
- $R^{coll} = q_k \cdot (V \cdot ELR)$
- $R^{GB}, R^{WN}, \& R^c$ are all credibility weighted estimates of reserves, using R^{ind} & R^{coll}
- $Z^{GB} = p$
- $Z^{WN} = p \cdot ELR$
- $Z^c = \frac{p}{p+\sqrt{p}}$
- R^c is optimal in the sense that it minimizes the mean squared error of the reserve estimate, under certain assumptions

Errata

Remark 6.1

Earlier in the paper, the author uses the variable f_i to represent the ratio of the estimate of variance of ultimate losses to the variance of the burning cost estimate:

$$f_i = \frac{\widehat{Var}(U_i)}{\widehat{Var}(U_i^{BC})}$$

In this section he uses f_k^{CL} to represent loss development factors.

These are two very different meanings of the variable name f.

The Hurlimann paper has a typo in table 7.4, in the Collective and Optimal Columns.
If you are using the manual, no need to worry about this.

Table 7.3 is correct, so it looks to me like the author simply added the wrong columns in a couple of places.

Table 7.3

Reserves

	Collective	Individual	Neuhaus	Benktander	Optimal
All	25,154	26,972	25,913	25,999	25,914
2	705	544	568	553	626
3	1,736	1,518	1,564	1,544	1,630
4	3,380	2,761	2,962	2,915	3,092
5	7,166	10,829	8,904	9,101	8,708
6	12,167	11,320	11,916	11,887	11,858

Table 7.4

Ultimate

	Collective	Collective (corrected)	Individual	Neuhaus	Benktander	Optimal	Optimal (corrected)
All	~~86,572~~	**85,992**	87,810	86,751	86,837	~~86,486~~	**86,752**
1	14,307	**14,307**	14,307	14,307	14,307	14,307	**14,307**
2	~~9,964~~	**10,043**	9,882	9,906	9,891	~~9,966~~	**9,964**
3	~~12,772~~	**12,878**	12,660	12,706	12,686	~~12,779~~	**12,772**
4	~~11,443~~	**11,731**	11,112	11,313	11,266	~~11,484~~	**11,443**
5	~~20,826~~	**19,284**	22,947	21,022	21,219	~~20,364~~	**20,826**
6	~~17,440~~	**17,749**	16,902	17,498	17,469	~~17,586~~	**17,440**

The CAS posted a correction. In the correction the Collective column is now correct. The Optimal column still has the incorrect figures.