

The Infinite Actuary Exam STAM Online Course
A.1.3. Practice Problems on Generating Functions

1. A discrete distribution N has probability generating function

$$P_N(z) = (0.3 + 0.2z + 0.5z^2)^5$$

Find $P[N = 2]$.

A. 0.023

B. 0.031

C. 0.046

D. 0.055

E. 0.062

2. A discrete distribution N has probability generating function

$$P_N(z) = (0.3 + 0.2z + 0.5z^2)^5$$

Find $\text{Var}[N]$.

A. 1.9

B. 3.8

C. 7.6

D. 33.8

E. 39.8

3. An actuary models the number of losses using a distribution with probability generating function

$$P(z) = 1 - (1 - z)^{1/4}, \quad z < 1$$

According to the model, what is the probability of having exactly 3 losses?

- A. $\frac{3}{64}$ B. $\frac{7}{128}$ C. $\frac{3}{16}$ D. $\frac{7}{32}$ E. $\frac{21}{64}$

4. [3.F06.25] You are given the following information about the probability generating function for a discrete distribution:

$$P'(1) = 2 \quad P''(1) = 6$$

Calculate the variance of the distribution.

- A. Less than 1.5
- B. At least 1.5, but less than 2.5
- C. At least 2.5, but less than 3.5
- D. At least 3.5, but less than 4.5
- E. At least 4.5

5. The moment generating function of X is $M_X(t) = e^{2t^2 - 5t}$. Find $\text{Var}[X]$.

A. 1

B. 2

C. 3

D. 4

E. 5

6. You are given that the probability generating function of a random variable X is

$$P_X(z) = \frac{1}{4 - 3z}$$

Find the second raw moment of X .

A. 3

B. 9

C. 12

D. 18

E. 21

7. An actuary models the number of losses using a Sibuya distribution with probability generating function

$$P(z) = 1 - (1 - z)^{1/3}, \quad z < 1$$

According to the model, what is the probability of having 2 losses?

- A. $1/18$ B. $1/9$ C. $1/6$ D. $2/9$ E. $1/3$