

The Infinite Actuary Exam STAM Online Course
A.1.3. Practice Problems on Generating Functions

1. A discrete distribution N has probability generating function

$$P_N(z) = (0.3 + 0.2z + 0.5z^2)^5$$

Find $P[N = 2]$.

- A. 0.023 B. 0.031 C. 0.046 D. 0.055 E. 0.062

2. A discrete distribution N has probability generating function

$$P_N(z) = (0.3 + 0.2z + 0.5z^2)^5$$

Find $\text{Var}[N]$.

- A. 1.9 B. 3.8 C. 7.6 D. 33.8 E. 39.8

3. An actuary models the number of losses using a distribution with probability generating function

$$P(z) = 1 - (1 - z)^{1/4}, \quad z < 1$$

According to the model, what is the probability of having exactly 3 losses?

- A. $\frac{3}{64}$ B. $\frac{7}{128}$ C. $\frac{3}{16}$ D. $\frac{7}{32}$ E. $\frac{21}{64}$

4. [3.F06.25] You are given the following information about the probability generating function for a discrete distribution:

$$P'(1) = 2 \quad P''(1) = 6$$

Calculate the variance of the distribution.

- A. Less than 1.5
B. At least 1.5, but less than 2.5
C. At least 2.5, but less than 3.5
D. At least 3.5, but less than 4.5
E. At least 4.5

5. The moment generating function of X is $M_X(t) = e^{2t^2 - 5t}$. Find $\text{Var}[X]$.

- A. 1 B. 2 C. 3 D. 4 E. 5

6. You are given that the probability generating function of a random variable X is

$$P_X(z) = \frac{1}{4 - 3z}$$

Find the second raw moment of X .

- A. 3 B. 9 C. 12 D. 18 E. 21

7. An actuary models the number of losses using a Sibuya distribution with probability generating function

$$P(z) = 1 - (1 - z)^{1/3}, \quad z < 1$$

According to the model, what is the probability of having 2 losses?

- A. 1/18 B. 1/9 C. 1/6 D. 2/9 E. 1/3