

The Infinite Actuary Exam STAM Online Course

A.1.7. Practice Problems on Sums

1. [4B.S93.9] If X and Y are independent random variables, which of the following statements are true?

- (i) $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$
- (ii) $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$
- (iii) $\text{Var}(aX + bY) = a^2\text{E}[X^2] - a(\text{E}[X])^2 + b^2\text{E}[Y^2] - b(\text{E}[Y])^2$

A. (i) only B. (i) and (ii) only C. (i) and (iii) only D. (ii) and (iii) only E. (i), (ii) and (iii)

2. An insurance company has two types of customers: high risk customers, whose annual loss amounts have mean 20 and variance 50, and low risk customers, whose annual losses have mean 10 and variance 20.

The loss amounts of the different customers are all independent. In a group with 6 high risk and 4 low risk customers, what is the variance of the total annual losses from the group?

A. 38 B. 62 C. 289 D. 380 E. 620

3. I have two fair, but unusual coins. One is gold, and the two sides are marked “1” and “3,” while the other is silver with its sides marked “2” and “4.” Suppose that I flip both coins. Let W be the average of the two sides that land face up. Let Z be the value that is face up if I choose one of the two coins at random to look at.

Let $a = \text{P}[Z = 1] - \text{P}[W = 1]$, and let $b = \text{Var}[Z] - \text{Var}[W]$. Find a and b .

- A. $a = 0, b = 0$
- B. $a = 0, b = 0.25$
- C. $a = 0.25, b = 0.25$
- D. $a = 0.25, b = 0.5$
- E. $a = 0.25, b = 0.75$

4. [4.F00.32] You are given the following for a sample of five observations from a bivariate distribution:

x	y
1	4
2	2
4	3
5	6
6	4

(i)

(ii) $\bar{x} = 3.6, \bar{y} = 3.8$

A is the covariance of the empirical distribution F_e as defined by these five observations. B is the maximum possible covariance of an empirical distribution with identical marginal distributions to F_e . Determine $B - A$.

A. 0.9 B. 1.0 C. 1.1 D. 1.2 E. 1.3

5. [4B.F99.29] You are given the following:

- A is a random variable with mean 5 and coefficient of variation 1.
- B is a random variable with mean 5 and coefficient of variation 1.
- C is a random variable with mean 20 and coefficient of variation $1/2$.
- A, B and C are independent.
- $X = A + B$
- $Y = A + C$

Determine the correlation coefficient between X and Y .

- A. $\frac{-2}{\sqrt{10}}$ B. $\frac{-1}{\sqrt{10}}$ C. 0 D. $\frac{1}{\sqrt{10}}$ E. $\frac{2}{\sqrt{10}}$

6. Let X and Y be independent Poissons with means 2 and 3, respectively. If $W = 0.4X + 0.6Y$, find $\text{Var}[W] - \text{E}[W]$.

- A. -1.2 B. -0.24 C. 0 D. 0.24 E. 1.2