

The Infinite Actuary Exam STAM Online Course

A.2.4. Practice Problems on Normal Distributions

1. The average height of adult Americans is 176 cm, with a standard deviation of 6 cm, for males, and 163 cm, with a standard deviation of 5 cm, for females. If heights of each group are normally distributed, what is the probability that a randomly selected American male is taller than a randomly selected American female?

A. 0.85

B. 0.88

C. 0.91

D. 0.93

E. 0.95

2. [3.F05.32] For a certain insurance company, 60% of claims have a normal distribution with mean 5,000 and variance 1,000,000. The remaining 40% have a normal distribution with mean 4,000 and variance 1,000,000.

Calculate the probability that a randomly selected claim exceeds 6,000.

- A. Less than 0.10
- B. At least 0.10, but less than 0.15
- C. At least 0.15, but less than 0.20
- D. At least 0.20, but less than 0.25
- E. At least 0.25

3. Suppose X and Y are independent normals, each with variance 1,000,000, and with $E[X] = 5,000$ and $E[Y] = 4,000$.

Find $P[0.6X + 0.4Y > 6,000]$

- A. 0.026 B. 0.052 C. 0.081 D. 0.104 E. 0.123

4. [1.S01.33] For Company A there is a 60% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 10,000 and standard deviation 2,000.

For Company B there is a 70% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 9,000 and standard deviation 2,000.

Assume that the total claim amounts of the two companies are independent.

What is the probability that, in the coming year, Company B's total claim amount will exceed Company A's total claim amount?

- A. 0.180 B. 0.185 C. 0.217 D. 0.223 E. 0.240

5. Loss amounts are normally distributed, with a 6.68% chance of exceeding 102 and a 2.28% chance of exceeding 105. What is the probability that a randomly selected loss is greater than 95?
- A. 0.11 B. 0.37 C. 0.48 D. 0.66 E. 0.91

6. Let Y be a mixture of X_1 and X_2 , where X_1 is a normal random variable with mean 0 and standard deviation 1, and X_2 is a normal random variable with mean 0 and standard deviation 5. If $P[Y = X_1] = 0.9$, what is the kurtosis of Y ?

Recall that the kurtosis of a normal random variable is 3.

- A. 3.0 B. 8.2 C. 12.4 D. 16.5 E. 49.5

7. Losses are lognormal with median 3 and mean 4. What is the variance of a randomly selected loss?

A. 0.6

B. 2.7

C. 4.5

D. 7.4

E. 12.4

8. Losses are lognormal with mean 3 and standard deviation 2. What is the probability that a loss that exceeds 1 will be greater than 4?

A. 0.23

B. 0.26

C. 0.37

D. 0.86

E. 0.89

9. [4B.F97.26] You are given the following:

- In 1996, losses follow a lognormal distribution with parameters μ and σ .
- In 1997, losses follow a lognormal distribution with parameters $\mu + \ln(k)$ and σ , where $k > 1$.
- In 1996, 100p% of the losses exceed the mean of the losses in 1997.

Determine σ . Note: z_p is the 100pth percentile of a normal distribution with mean 0 and variance 1.

A. $2 \ln(k)$ B. $-z_p \pm \sqrt{z_p^2 - 2 \ln(k)}$ C. $z_p \pm \sqrt{z_p^2 - 2 \ln(k)}$ D. $\sqrt{-z_p \pm \sqrt{z_p^2 - 2 \ln(k)}}$ E. $\sqrt{z_p \pm \sqrt{z_p^2 - 2 \ln(k)}}$

10. Loss amounts have a survival function given by

$$S(x) = \begin{cases} 1 & x < 0 \\ e^{-2x^2} & x \geq 0 \end{cases}$$

What is the average loss amount?

A. $\sqrt{\frac{\pi}{8}}$

B. $\sqrt{\frac{\pi}{4}}$

C. $\sqrt{\frac{\pi}{2}}$

D. $\sqrt{\pi}$

E. $\sqrt{2\pi}$

11. Loss amounts have a survival function given by

$$S(x) = \begin{cases} 1 & x < 1 \\ e^{-2x^2} & x \geq 1 \end{cases}$$

What is the average loss amount?

A. 0.03

B. 0.77

C. 1.03

D. 1.77

E. 2.41

12. The density function of a random variable is proportional to e^{-x^2} for $x \geq 0.5$ and is 0 otherwise. Find $P[X \geq 1]$

A. 0.08

B. 0.33

C. 0.55

D. 0.76

E. 0.92

13. Losses are modeled with a lognormal distribution with parameters μ and σ . If the median loss amount is 1.06 and the mean loss amount is 1.08, what is σ ?
- A. 0.01 B. 0.02 C. 0.04 D. 0.10 E. 0.19

14. Losses are modeled with a lognormal distribution with mean 0.42 and variance 1.65. Find the probability that losses are at least 1.
- A. 0.09 B. 0.18 C. 0.27 D. 0.33 E. 0.36

15. Suppose that X and Y are jointly normal random variables, with $E[X] = 1$, $\text{Var}[X] = 4$, and $E[Y] = -2$, $E(Y^2) = 5$. If the correlation of X and Y is $-1/2$, what is the probability that the sum of X and Y is positive?

A. 0.16

B. 0.28

C. 0.37

D. 0.72

E. 0.84

16. Variant on [3-CAS.F05.32] Seventy-five percent of claims have a normal distribution with a mean of 3,000 and a variance of 1,000,000. The remaining 25% have a normal distribution with a mean of 4,000 and a variance of 1,000,000. Determine the probability that a randomly chosen claim exceeds 5,000.
- A. Less than 0.040
 - B. At least 0.040, but less than 0.045
 - C. At least 0.045, but less than 0.050
 - D. At least 0.050, but less than 0.055
 - E. At least 0.055

17. Loss amounts X have a lognormal distribution with parameters $\mu = 2$ and $\sigma^2 = 0.64$. What is the skewness of X ?
- A. 0.4 B. 3.7 C. 6.8 D. 10.2 E. 14.1