

The Infinite Actuary Exam STAM Online Course

A.2.5. Uniform Distributions

Last updated April 11, 2018

1. [4B.F95.28] Two numbers are drawn independently from a uniform distribution on $[0, 1]$. What is the variance of their product?

A. 1/144 B. 3/144 C. 4/144 D. 7/144 E. 9/144

Let X and Y be the two numbers.

$$\begin{aligned} E[X] &= E[Y] = \frac{1}{2} \\ E[X^2] &= E[Y^2] = \frac{1}{12} + \left(\frac{1}{2}\right)^2 = \frac{1}{3} \\ \text{Var}[XY] &= E[(XY)^2] - (E[XY])^2 \\ &= E[X^2Y^2] - (E[X]E[Y])^2 \\ &= E[X^2]E[Y^2] - (E[X])^2(E[Y])^2 \\ &= \frac{1}{3} \cdot \frac{1}{3} - \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^2 \\ &= \boxed{\frac{7}{144}} \end{aligned}$$

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2. Suppose that X is an exponential random variable with mean θ , where θ is a continuous uniform on $(0, 2)$.

Find the mean of X .

A. 0.5 B. 1.0 C. 1.5 D. 2.0 E. 2.5

$$E[X] = E[E[X \mid \theta]] = E[\theta] = \frac{2+0}{2} = \boxed{1}$$

3. Suppose that X is an exponential random variable with mean θ , where θ is a continuous uniform on $(0, 2)$.

Find the variance of X .

A. 1 B. 4/3 C. 5/3 D. 6/3 E. 7/3

Using the conditional variance formula,

$$\begin{aligned} \text{Var}(X) &= E[\text{Var}(X \mid \theta)] + \text{Var}[E(X \mid \theta)] \\ &= E[\theta^2] + \text{Var}[\theta] \\ &= \left(\frac{(2-0)^2}{12} + \left(\frac{2+0}{2} \right)^2 \right) + \left(\frac{(2-0)^2}{12} \right) \end{aligned}$$

$$= \frac{4}{3} + \frac{1}{3} = \boxed{\frac{5}{3}}$$
