Review - Outline

1. A.1.1 Review
   - Continuously Compounded Returns

Interest Rate Compounding

If r is quoted as an **effective** annual interest rate, then if you invest $\$X$ today, in t years you will have $X(1+r)^t$

If r is quoted as a **continuously compounded** annual interest rate, then if you invest $\$X$ today, in t years you will have Xe^{rt}

If you purchase asset S at time t for price S_t and sell it for price S_{t+h} in the future, then your continuously compounded return on the investment for period h must solve:

$$\begin{aligned} S_t e^r &= S_{t+h} \\ r &= \ln(S_{t+h}/S_t) \end{aligned}$$

Return Volatility

An asset's **volatility**, σ, generally refers to the sample standard deviation of its returns. Given returns $r_1, r_2, r_3, ..., r_N$, volatility can be calulated as:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(r_i - \bar{r})^2}$$

Note that:

- The formula inside the radical gives the sample variance, σ^2
- $\bar{r}$ is the sample average. I.e., $\bar{r} = \dfrac{1}{N}\sum_{i=1}^{N} r_i$

Properties of Continuously Compounded Returns

Let $r_h, r_{2h}, r_{3h}, \ldots, r_{nh}$ be the continuously compounded returns measured at frequency h on asset S between times 0 and T, where $h = T/n$ is measured in years. Then:

1. Increases and decreases are symmetric
   - If $r_h = R$ and $r_{2h} = -R$, then $S_{2h} = S_0 e^R e^{-R} = S_0$
2. Returns are additive
   - $\ln\left(\dfrac{S_T}{S_0}\right) = \displaystyle\sum_{i=1}^{n} r_{ih}$
3. Volatility is proportional to the square root of time
   - E.g., let σ_h be the volatility of the returns measured at frequency h. Then $\sigma = \dfrac{\sigma_h}{\sqrt{h}}$, where σ is the annual volatility.
   - This implies variance is proportional to time

The following table gives the month-end prices of a non-dividend paying stock for five consecutive months:

Month	Price
Jan	34
Feb	31
Mar	34
Apr	36
May	37

Estimate the annual volatility for this stock.

A. 7% B. 8% C. 16% D. 24% E. 28%

1. Use calculator to calculate the cont. compounded returns
2. Use the stat menu to find $\sigma_{monthly} = .0802$
3. Annualize the volatility

$$\sigma_{annual} = \sigma_{monthly} \times \sqrt{12} = 0.2778$$