A.2.2 Actuarial Notation for Probabilities

${}_tp_x$

${}_tp_x$ – probability that (x) will attain age $x+t$.

$$ {}_tp_x = \Pr(T_x > t) = S_x(t) $$

If $t = 1$, prefix omitted

$$ p_x = \text{probability } (x) \text{ will attain age } x+1 $$

Useful relationship

$$ {}_{t+u}p_x = {}_tp_x \cdot {}_up_{x+t} $$

${}_tq_x$

${}_tq_x$ – probability that (x) dies within t years.

$$ {}_tq_x = \Pr(T_x \leqslant t) = F_x(t)$$

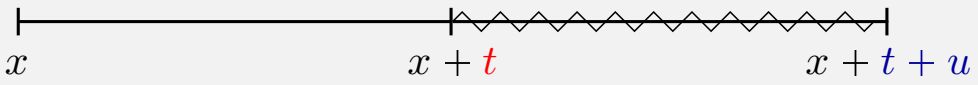
$$ {}_tq_x = 1 - {}_tp_x$$

If $t = 1$, prefix omitted

$$q_x = \text{probability } (x) \text{ dies within 1 year}$$
$$\text{aka mortality rate for age } x$$

${}_{t|u}q_x$

${}_{t|u}q_x$ – probability that (x) will survive t years and die within the following u years.



$$ {}_{t|u}q_x = \Pr(t < T_x \leqslant t+u) = S_x(t) - S_x(t+u)$$

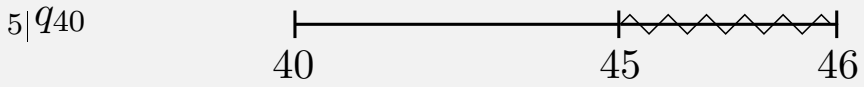
$$ = {}_tp_x - {}_{t+u}p_x$$

$$ = {}_{t+u}q_x - {}_tq_x$$

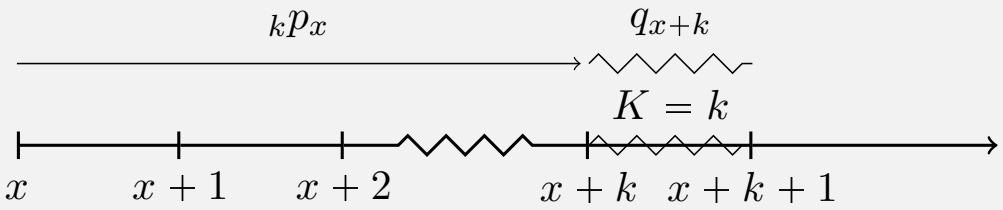
$$ = {}_tp_x \cdot {}_uq_{x+t}$$

${}_{t|u}q_x$ cont.

If $u = 1$, prefix omitted



Revisit Curtate Future Lifetime



$$\Pr(K_x = k) = {}_kp_x \cdot q_{x+k} = {}_{k|}q_x$$

$\therefore {}_{k|}q_x$ is PDF for curtate future lifetime for (x)

$$\sum_{k=0}^{\infty} {}_{k|}q_x = 1 \qquad F_{K_x}(y) = \sum_{h=0}^{\lfloor y \rfloor} {}_{h|}q_x$$

Summary of Actuarial Notation

1. ${}_tp_x$
   - probability (x) survives at least t years
   - $S_x(t)$, survival function for T_x
2. ${}_tq_x$
   - probability (x) dies before age $x+t$
   - $F_x(t)$, cumulative distribution function for T_x
3. ${}_{t|u}q_x$
   - probability (x) survives t years and dies in the following u years
   - ${}_{t|}q_x$ is the probability density function for K_x

Exercise 1

Describe the following in words:

1. ${}_{10}p_{30}$
2. ${}_tq_x$
3. ${}_{5|}q_{40}$

Exercise 1

Describe the following in words:

1. ${}_{10}p_{30}$
2. ${}_{t}q_{x}$
3. ${}_{5|}q_{40}$

Answers:

1. ${}_{10}p_{30}$ - probability (30) survives to age 40
2. ${}_{t}q_{x}$ - probability (x) dies before age $x+t$
3. ${}_{5|}q_{40}$ - probability (40) dies between ages 45 and 46

Exercise 2

You are given ${}_{1|}q_{x+1} = 0.095$, ${}_{2|}q_{x+1} = 0.171$ and $q_{x+3} = 0.200$. Calculate $q_{x+1} + q_{x+2}$.

Exercise 2

You are given ${}_{1|}q_{x+1} = 0.095$, ${}_{2|}q_{x+1} = 0.171$ and $q_{x+3} = 0.200$. Calculate $q_{x+1} + q_{x+2}$.

Problem solving technique – write out every relationship for the given information that you can think of:

- ${}_{1|}q_{x+1} = p_{x+1} \cdot q_{x+2}$ — 3 - finally find q_{x+2}
  $= p_{x+1} - {}_2p_{x+1}$ — 2 - plug into here to find q_{x+1}
  $= {}_2q_{x+1} - q_{x+1}$
- ${}_{2|}q_{x+1} = {}_2p_{x+1} \cdot q_{x+3}$ — 1 - start here b/c one unknown
  $= {}_2p_{x+1} - {}_3p_{x+1}$
  $= {}_3q_{x+1} - {}_2q_{x+1}$

Exercise 2

You are given ${}_{1|}q_{x+1} = 0.095$, ${}_{2|}q_{x+1} = 0.171$ and $q_{x+3} = 0.200$. Calculate $q_{x+1} + q_{x+2}$.

Step 1 - Find ${}_2p_{x+1}$

$${}_{2|}q_{x+1} = {}_2p_{x+1} \cdot q_{x+3}$$
$$0.171 = {}_2p_{x+1} \cdot 0.2$$
$${}_2p_{x+1} = 0.855$$

Step 2 - Find q_{x+1}

$${}_{1|}q_{x+1} = p_{x+1} - {}_2p_{x+1}$$
$$0.095 = p_{x+1} - 0.855$$
$$p_{x+1} = 0.95$$
$$q_{x+1} = 0.05$$

Step 3 - Find q_{x+2}

$${}_2p_{x+1} = 0.855$$
$$p_{x+1} \cdot p_{x+2} = 0.855$$
$$0.95 \cdot p_{x+2} = 0.855$$
$$p_{x+2} = 0.9$$
$$q_{x+2} = 0.1$$

$$q_{x+1} + q_{x+2} = 0.05 + 0.1 = \boxed{0.15}$$

Exercise 3

Prove that ${}_{t+u}p_x = {}_tp_x \cdot {}_up_{x+t}$.

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Prove that ${}_{t+u}p_x = {}_tp_x \cdot {}_up_{x+t}$.

$$
\begin{aligned}
{}_{t+u}p_x &= \Pr(T_x > t+u) \\
&= \Pr(T_0 > x+t+u \,|\, T_0 > x) \\
&= \frac{\Pr(T_0 > x+t+u)}{\Pr(T_0 > x)} \cdot \frac{\Pr(T_0 > x+t)}{\Pr(T_0 > x+t)} \\
&= \frac{\Pr(T_0 > x+t)}{\Pr(T_0 > x)} \cdot \frac{\Pr(T_0 > x+t+u)}{\Pr(T_0 > x+t)} \\
&= \Pr(T_0 > x+t \,|\, T_0 > x) \cdot \Pr(T_0 > x+t+u \,|\, T_0 > x+t) \\
&= {}_tp_x \cdot {}_up_{x+t} \quad \checkmark
\end{aligned}
$$

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