

Last updated April 1, 2019.

1. [First Pass] Which of the following are true?

1.  ${}_t|{}_uq_x = {}_tp_x \cdot {}_uq_{x+t}$

2.  ${}_t|{}_uq_x = \frac{{}_l{}_{x+t+u} - {}_l{}_{x+t}}{{}_l{}_x}$

3.  ${}_t|{}_uq_x = {}_tp_x - {}_{t+u}p_x$

A. 1

B. 2

C. 3

D. 1, 2

E. 1, 3

2. [First Pass] Given that a life aged 50 will live to age 60, what is the probability  $p$  that he will die between ages 70 and 80?

Age	$l_x$
50	89,509
60	81,881
70	66,162
80	39,144

A. Less than 0.310

B. At least 0.310, but less than 0.315

C. At least 0.315, but less than 0.320

D. At least 0.320, but less than 0.325

E. At least 0.325

3. [First Pass] You are given the following mortality table:

Age( $x$ )	$q_x$	$l_x$	$d_x$
20		30,000	1,200
21			
22		27,350	
23	0.0700		
24	0.0790	23,900	

Determine the probability that a life aged 21 will die within two years.

- A. Less than 0.0960  
B. At least 0.0960, but less than 0.1010  
C. At least 0.1010, but less than 0.1060  
D. At least 0.1060, but less than 0.1110  
E. At least 0.1110
4. [First Pass] Given the following portion of a life table:

$x$	$l_x$	$d_x$	$p_x$	$q_x$
0	1,000		0.875	
1				
2	750			0.25
3				
4				
5	200	120		
6				
7		20		1.00

Determine the value of  $p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot q_6$ .

- A. Less than 0.055  
B. At least 0.055, but less than 0.065  
C. At least 0.065, but less than 0.075  
D. At least 0.075  
E. The answer cannot be determined from the given information.

5. [First Pass] You are given  $S_0(x) = \frac{1}{1+x}$ .

Determine the median future lifetime of  $(y)$ .

- A.  $y + 1$                       B.  $y$                       C. 1                      D.  $\frac{1}{y}$                       E.  $\frac{1}{1+y}$

6. You are given the following information:

(i)  $l_1 = 9700$

(ii)  $q_1 = q_2 = 0.020$

(iii)  $q_4 = 0.026$

(iv)  $d_3 = 232$

Determine the expected number of survivors to age 5.

- A. Less than 8,845  
B. At least 8,845, but less than 8,850  
C. At least 8,850, but less than 8,855  
D. At least 8,855, but less than 8,860  
E. At least 8,860

7. You are given the following information:

- (i) The probability that two 70-year-olds are both alive in 20 years is 16%.
- (ii) The probability that two 80-year-olds are both alive in 20 years is 1%.
- (iii) There is an 8% chance of a 70-year-old living 30 years.
- (iv) All lives are independent and have the same expected mortality.

Determine the probability of an 80-year-old living 10 years.

- A. Less than 0.35
- B. At least 0.35, but less than 0.45
- C. At least 0.45, but less than 0.55
- D. At least 0.55, but less than 0.65
- E. At least 0.65

8. Light bulbs burn out according to the following life table:

$x$	$l_x$
0	1,000,000
1	800,000
2	600,000
3	300,000
4	0

A new plant has 2,500 light bulbs. Burned out light bulbs are replaced with new light bulbs at the end of each year.

What is the expected number of new light bulbs that will be needed at the end of year 3?

- A. Less than 800
- B. At least 800, but less than 860
- C. At least 860, but less than 920
- D. At least 920, but less than 980
- E. At least 980

9. [SOA.MLC.200] The graph of a piecewise linear survival function,  $S_0(t)$ , consists of 3 line segments with endpoints  $(0, 1)$ ,  $(25, 0.50)$ ,  $(75, 0.40)$ ,  $(100, 0)$ .

Calculate  $\frac{{}_{20|55}q_{15}}{{}_{55}q_{35}}$ .

- A. 0.69                      B. 0.71                      C. 0.73                      D. 0.75                      E. 0.77

10. [3.S00.28] For a mortality study on college students:

- (i) Students entered the study on their birthdays in 1963.
- (ii) You have no information about mortality before birthdays in 1963.
- (iii) Dick, who turned 20 in 1963, died between his 32nd and 33rd birthdays.
- (iv) Jane, who turned 21 in 1963, was alive on her birthday in 1998, at which time she left the study.
- (v) All lifetimes are independent.
- (vi) Likelihoods are based upon the Standard Ultimate Life Table.

Calculate the likelihood for these two students.

- A. 0.00029                      B. 0.00033                      C. 0.00039                      D. 0.00043                      E. 0.00049

11. [SOA 3.4; adapted from MLC.F13.24] The SULT Club has 4000 members all age 25 with independent future lifetimes. The mortality for each member follows the Standard Ultimate Life Table.

Calculate the largest  $N$ , using the normal approximation, such that the probability that there are at least  $N$  survivors at age 95 is at least 90%.

- A. 800                      B. 815                      C. 830                      D. 845                      E. 860

12. [SOA 2.2; MLC.S13.20] Scientists are searching for a vaccine for a disease. You are given:

- (i) 100,000 lives age  $x$  are exposed to the disease.
- (ii) Future lifetimes are independent, except that the vaccine, if available, will be given to all at the end of year 1.
- (iii) The probability that the vaccine will be available is 0.2.
- (iv) For each life during year 1,  $q_x = 0.02$ .
- (v) For each life during year 2,  $q_{x+1} = 0.01$  if the vaccine has been given, and  $q_{x+1} = 0.02$  if it has not been given.

Calculate the standard deviation of the number of survivors at the end of year 2.

- A. 100                      B. 200                      C. 300                      D. 400                      E. 500

13. [SOA 3.10; MLC.F15.02] A group of 100 people start a Scissor Usage Support Group. The rate at which members enter and leave the group is dependent on whether they are right-handed or left-handed.

You are given the following:

- (i) The initial membership is made up of 75% left-handed members (L) and 25% right-handed members (R).
- (ii) After the group initially forms, 35 new (L) and 15 new (R) join the group at the start of each subsequent year.
- (iii) Members leave the group only at the end of each year.
- (iv)  $q^L = 0.25$  for all years.
- (v)  $q^R = 0.50$  for all years.

Calculate the proportion of the Scissor Usage Support Group's expected membership that is left-handed at the start of the group's 6<sup>th</sup> year, before any new members join for that year.

- A. 0.76                      B. 0.81                      C. 0.86                      D. 0.91                      E. 0.96