Last updated August 24, 2018.

1. [First Pass] You are given $S_0(x) = e^{-\frac{x^3}{12}}$, for $x \geq 0$.

   Determine μ_x.

   A. $-\frac{x^2}{4}$ B. $1 - \frac{x^2}{4}$ C. $\frac{x^2}{4}$ D. $\frac{x^2}{4} e^{-\frac{x^2}{12}}$ E. $-\frac{x^3}{12}$

$$\begin{aligned}
\mu_x &= -\frac{S_0'(x)}{S_0(x)} \\
&= -\frac{\exp\left(-\frac{x^3}{12}\right) \cdot \left(-\frac{3x^2}{12}\right)}{\exp\left(-\frac{x^3}{12}\right)} \\
&= \boxed{\frac{x^2}{4}}
\end{aligned}$$

---

2. [First Pass] You are given a life, aged 30, subject to a force of mortality given by:

   $\mu_x = 0.02 \cdot x^{0.5}$, for $20 \leq x \leq 50$.

   Determine the probability this life will survive 5 years and die during the following year.

   A. Less than 0.044
   B. At least 0.044, but less than 0.052
   C. At least 0.052, but less than 0.060
   D. At least 0.060, but less than 0.068
   E. At least 0.068

$$\begin{aligned}
{}_{5|}q_{30} &= {}_5p_{30} - {}_6p_{30} \\
&= \exp\left(-\int_{30}^{35} 0.02\, x^{0.5}\, dx\right) - \exp\left(-\int_{30}^{36} 0.02\, x^{0.5}\, dx\right) \\
&= \exp\left(-\frac{0.02}{1.5} x^{1.5}\Bigg|_{30}^{35}\right) - \exp\left(-\frac{0.02}{1.5} x^{1.5}\Bigg|_{30}^{36}\right) \\
&= 0.5656 - 0.5020
\end{aligned}$$

$$= \boxed{0.0636}$$

---

3. [First Pass] Which of the following functions can serve as a force of mortality?

1.	Bc^x	$B > 0, 0 < c < 1, x \geq 0$
2.	$B(x+1)^{-0.5}$	$B > 0, x \geq 0$
3.	$k(x+1)^n$	$k > 0, n > 0, x \geq 0$

A. 1 and 2 only B. 1 and 3 only C. 2 and 3 only D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

---

1. $S_0(x) = e^{-\int_0^x Bc^t\,dt} = e^{-\frac{Bc^t}{\ln c}\Big|_0^x} = e^{\frac{B}{\ln c} - \frac{Bc^x}{\ln c}}$

   $S_0(\infty) = e^{\frac{B}{\ln c}} \neq 0$

2. $S_0(x) = e^{-\int_0^x B(t+1)^{-0.5}\,dt} = e^{-2B(t+1)^{0.5}\Big|_0^x} = e^{2B-2B(x+1)^{0.5}}$

   $S_0(\infty) = e^{-\infty} = 0$ ✓

3. $S_0(x) = e^{-\int_0^x k(t+1)^n\,dt} = e^{-\frac{k(t+1)^{n+1}}{n+1}\Big|_0^x} = e^{\frac{k-k(x+1)^{n+1}}{n+1}}$

   $S_0(\infty) = e^{-\infty} = 0$ ✓

Thus, $\boxed{\text{2 and 3 only}}$.

---

4. [SOA.MLC.155] Given:

   (i) $\mu_x = F + e^{2x}, \quad x \geq 0$

   (ii) ${}_{0.4}p_0 = 0.50$

   Calculate F.

   A. -0.20 B. -0.09 C. 0.00 D. 0.09 E. 0.20

Add up the force of mortality between ages 0 and 0.4, take the negative and take the exponential

$$
\begin{aligned}
{}_{0.4}p_0 &= \exp\left(-\int_0^{0.4} F + e^{2t}\, dt\right) \\
0.5 &= \exp\left(-\left[Ft + \frac{e^{2t}}{2}\right]_0^{0.4}\right) \\
0.5 &= \exp\left(-0.4F - \frac{e^{0.8}}{2} + \frac{1}{2}\right) \\
-\ln 0.5 &= 0.4F + \frac{e^{0.8}}{2} - \frac{1}{2} \\
F &= \boxed{0.20}
\end{aligned}
$$

---

5. [SOA.MLC.032] Given: The survival function $S_0(t)$, where

$$
S_0(t) = \begin{cases} 1 & 0 \le t < 1 \\ 1 - \frac{e^t}{100} & 1 \le t < 4.5 \\ 0 & 4.5 \le t \end{cases}
$$

Calculate μ_4.

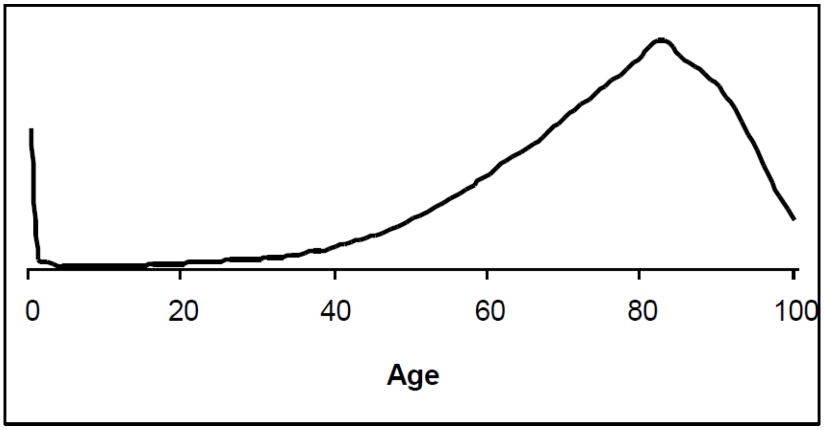
A. 0.45 B. 0.55 C. 0.80 D. 1.00 E. 1.20

$$
\mu_x = \frac{-S_0'(x)}{S_0(x)} = \frac{\frac{e^x}{100}}{1 - \frac{e^x}{100}}
$$

$$
\mu_4 = \frac{\frac{e^4}{100}}{1 - \frac{e^4}{100}} = \boxed{1.20}
$$

---

6. [SOA.MLC.106] The following graph is related to current human mortality:



Which of the following functions of age does the graph most likely show?

A. μ_x B. $l_x\mu_x$ C. $l_x p_x$ D. l_x E. l_x^2

This is the curve of deaths $\boxed{\ell_x\, \mu(x)}$