Last updated June 5, 2019.

1. [First Pass] For a life aged 50, the curtate-expectation of life $e_{50} = 20$. For that same life, you are also given that $p_{50} = 0.97$.

   Determine e_{51}.

   A. Less than 18.75
   B. At least 18.75, but less than 19.00
   C. At least 19.00, but less than 19.25
   D. At least 19.25, but less than 19.50
   E. At least 19.50

Simple recursion

$$e_{50} = p_{50}(1 + e_{51})$$

$$20 = 0.97(1 + e_{51})$$

$$e_{51} = \boxed{19.619}$$

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2. [First Pass] You are given:

x	l_x
96	180
97	130
98	73
99	31
100	0

   Define K to be the curtate future lifetime of (96). Calculate $\mathrm{Var}(K)$.

   A. 0.39 B. 0.53 C. 0.91 D. 1.11 E. 1.50

K is the number of complete future birthdays of a life age 96. Completing the table we have

x	d_x	K
96	50	0
97	57	1
98	42	2
99	31	3
100	0	4

Now this is a 1/P problem

$$\begin{aligned}
\mathrm{E}(K) &= \frac{57}{180}(1) + \frac{42}{180}(2) + \frac{31}{180}(3) = 1.3 \\
\mathrm{E}(K^2) &= \frac{57}{180}(1)^2 + \frac{42}{180}(2)^2 + \frac{31}{180}(3)^2 = 2.8 \\
\mathrm{Var}(K) &= 2.8 - 1.3^2 = \boxed{1.11}
\end{aligned}$$

Don't forget you can use the TI-30XS Multi-view's Data and Stat function to find the variance once you have your table built.

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3. [SOA.MLC.021; 3-SOA.F03.28] For (x):

- K is the curtate future lifetime random variable.
- $q_{x+k} = 0.1(k+1)$, $k = 0, 1, 2, \ldots, 9$
- $X = \min(K, 3)$

Calculate $\mathrm{Var}(X)$.

A. 1.1 B. 1.2 C. 1.3 D. 1.4 E. 1.5

Just list all the possible values for $K \wedge 3$ and the corresponding probability

K	$K \wedge 3$	Prob
0	0	0.1
1	1	$0.9(0.2) = 0.18$
2	2	$0.9(0.8)(0.3) = 0.216$
3+	3	$1 - (0.1 + 0.18 + 0.216) = 0.504$

$$\mathrm{E}[K \wedge 3] = 1(0.18) + 2(0.216) + 3(0.504) = 2.124$$

$$\mathrm{E}[(K \wedge 3)^2] = 1^2(0.18) + 2^2(0.216) + 3^2(0.504) = 5.58$$

$$\mathrm{Var}[K \wedge 3] = 5.58 - 2.124^2 = \boxed{1.069}$$

Don't forget you can use the TI-30XS MultiView's Data and Stat function to find the variance once you have your table built.

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4. [SOA.MLC.145; 3.F00.25] Given:

- Superscripts M and N identify two forces of mortality and the curtate expectations of life calculated from them.
- $\mu_{25+t}^{N} = \begin{cases} \mu_{25+t}^{M} + 0.1(1-t) & 0 \le t \le 1 \\ \mu_{25+t}^{M} & t > 1 \end{cases}$
- $e_{25}^{M} = 10.0$

Calculate e_{25}^{N}.

A. 9.2 B. 9.3 C. 9.4 D. 9.5 E. 9.6

Note that a life aged 26 of type N experiences the same force of mortality as a life aged 26 of type M. Hence, $K_{26}^{N} \equiv K_{26}^{M}$, which implies $e_{26}^{N} = e_{26}^{M}$.

$$\begin{aligned}
\frac{e_{25}^{N}}{e_{25}^{M}} &= \frac{p_{25}^{N}\,\left(1 + e_{26}^{N}\right)}{p_{25}^{M}\,\left(1 + e_{26}^{M}\right)} \\
&= \frac{\exp\left(-\int_0^1 \mu_{25+t}^{M} + 0.1(1-t)\,dt\right)}{\exp\left(-\int_0^1 \mu_{25+t}^{M}\,dt\right)} \\
&= \exp\left(-\int_0^1 0.1(1-t)\,dt\right) \\
&= \exp\left(\frac{0.1}{2}(1-t)^2\Big|_0^1\right) \\
&= \exp\left(-\frac{0.1}{2}\right) \\
e_{25}^{N} &= 10\exp\left(-\frac{0.1}{2}\right) = \boxed{9.5123}
\end{aligned}$$