

There is a lot of confusion about when to use “borrow $PV(K)$ ” and when to use “lend $PV(K)$ ”. Let me try to clear that up.

If we start with a long position in an asset, that is similar to a purchased call option in that we want the price of the asset to go up. However the call option does not have the downside risk of a long position. To equate the two we add a purchased put option to the long position. These two positions are equivalent in terms of profit:

$$\text{long position} + \text{purchased put} = \text{purchased call}$$

In terms of cash flow these are clearly not equivalent. At time 0 the left hand side will have a relatively large cash outflow to buy the asset and the put option while the right hand side will have a much smaller cash outflow. To offset the large cash outflow we borrow $PV(K)$.

$$\text{long position} + \text{purchased put} + \text{borrow } PV(K) = \text{purchased call}$$

A simple table showing the cash flows for the two equations can prove the above equation

Equation Side		CASH FLOW		
		Time 0	Time T	
			$S_T < K$	$S_T > K$
left	long position	$-S_0$	$+S_T$	$+S_T$
	purchased put	$-\text{Put}(K, T)$	$K - S_T$	0
	borrow $PV(K)$	$+Ke^{-rT}$	$-K$	$-K$
	total	$-S_0 - \text{Put}(K, T) + Ke^{-rT}$	0	$S_T - K$
right	purchased call	$-\text{Call}(K, T)$	0	$S_T - K$

The “left” and “right” hand sides have the same cash flows at time T , therefore they must have the same cash flows at time 0 (otherwise arbitrage would be possible). Since they have the same cash flows at all times, then they are equivalent positions and have the same profit.

Because the cash flows at time 0 must be equal, we can use this to prove

put-call parity.

$$-S_0 - \text{Put}(K, T) + Ke^{-rT} = -\text{Call}(K, T)$$

$$\text{Call}(K, T) - \text{Put}(K, T) = S_0 - Ke^{-rT}$$

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T}) - \text{PV}(K)$$

$$\text{Call}(K, T) - \text{Put}(K, T) = \text{PV}(F_{0,T} - K) \checkmark$$

The same can be done for the following equation:

$$\text{short position} + \text{purchased call} + \text{lend PV}(K) = \text{purchased put}$$

Equation Side		CASH FLOW		
		Time 0	Time T	
			$S_T < K$	$S_T > K$
left	short position	$+S_0$	$-S_T$	$-S_T$
	purchased call	$-\text{Call}(K, T)$	0	$S_T - K$
	lend $\text{PV}(K)$	$-Ke^{-rT}$	$+K$	$+K$
	total	$S_0 - \text{Call}(K, T) - Ke^{-rT}$	$K - S_T$	0
right	purchased put	$-\text{Put}(K, T)$	$K - S_T$	0

Again, the “left” and “right” hand sides have the same cash flows at time T , therefore they must have the same cash flows at time 0 (otherwise arbitrage would be possible). Since they have the same cash flows at all times, then they are equivalent positions and have the same profit.