# QFI Advanced Sample Flash Cards

You have downloaded a sample of our QFI Advanced flash cards. The flash cards are designed to help you memorize key material for the QFI Advanced exam.

The flash cards are in a “Q&A” format that is well-suited for reviewing the material at a high level after you complete section of the online seminar. The cards are sequenced in exactly the same order as the rest of the online seminar. Practicing your ability to recall the material in the form of an answer to a question is a great way to get ready for the actual exam.

We provide the PDF flash cards in two formats:

1. **“Singles”**. This version contains alternating front/back sides of each card in sequence. This format is well suited for PDF viewers on your computer, tablet, or phone. Simply flip through the pages.

2. **“FrontBack”**. This version has 3 cards per page. If you print this PDF double-sided on U.S. Letter (8.5” x 11”) paper, the front and back of each card will be aligned. This format also works well on Avery 5388 3x5” index cards, which can be purchased on Amazon. Additional printing instructions are included in the online seminar.

The sample that follows includes some of each format.

If you have any questions, email me anytime.

Zak Fischer, FSA, CERA

zak@theinfiniteactuary.com

State three alternatives to stochastic models.

1. Stress testing and scenario testing
2. Apply static load factors to deterministic assumptions
3. Develop ranges around best estimates

Implementation Steps for Stochastic Models

# Source: QFIA-124 (Part I)

1. Describe goals and intended uses
2. Consider using alternatives (stop here if not using stochastic)
3. Pick a projection technique
4. Decide on risk metrics
5. Establish which factors to model stochastically
6. Parameterize and fit distributions
7. Determine number of scenarios
8. Calibrate model
9. Run
10. Validate model and review output
11. Peer review
12. Communicate results

Risk-Neutral Scenarios

Source: QFIA-124 (Part I)

Price = Average PV of Risk-Neutral Scenarios at **Risk-Free Rate**

- No arbitrage (market consistent)
- Cash flows are risk-adjusted before discounting
- Scenarios are draws from a random distribution (e.g. normal)
- Must calibrate volatility to **current market data**
- Does not depend on historical returns

**Uses of Risk-Neutral Scenarios**

- Short-term, valuation/pricing focus
- Market-consistent insurance liabilities (e.g. FAS 133)
- Derivative pricing

Real-World Scenarios

## Source: QFIA-124 (Part I)

Price = Average PV of Real-World Scenarios at **Risky Rates**

- Rates = risk-free rate + risk premium
- Cash flows are realistic (not risk-adjusted)
- Reflect “stylized facts” about the real world
  - Equities yield more than risk-free rates long-term
  - Longer bonds yield more than shorter bonds
  - Yields increase with default risk (credit spreads)
  - Implied option volatilities $>$ equity volatilities

**Uses of Real-World Scenarios**

- Longer-term, projection/forecasting focus
- Create distributions of outcomes (profits, capital)
- Pricing to achieve outcome X% of the time
- Worst-case planning

Generating Interest Rate Scenarios

Source: QFIA-124 (Part II)

- **Maximum Likelihood Analysis**
  - Useful for determining distribution
  - Often suggests normal
- **Principal Component Analysis (PCA)**
  - Relates entire yield curve to just a few factors
  - Factors may be 3 key rates
- **HJM/BGM Arbitrage-Free Framework**
  - Risk-neutral approach

$$F_t = F_{t-1} + \text{Normal Random Change}_t$$

$$\text{Value} = \mathrm{E}\left[e^{-F_0-F_1-F_2-\cdots-F_N} CF_N\right]$$

- **Modifications for Long-Term, Realistic Scenarios**
  - Scenarios should tend toward historical yield curve shape
  - Apply caps and floors
- **Calibration**
  - Scale volatility
  - Validate mean reversion and slope

Interest Rate Parity

# Source: QFIA-124 (Part II)

Exchange rates vary with risk-free foreign and domestic rates

$$X_{t+1} = X_t \, e^{r_F - r_D}$$

Lognormal FX Model

# Source: QFIA-124 (Part II)

$$X_{t+\Delta t} = X_t \, e^{(r_F - r_D - \frac{\sigma^2}{2})\Delta t + \sigma Z \sqrt{\Delta t}}$$

- $r_F$ and $r_D$ can be deterministic or stochastic
- Stochastic versions are more complex (but more accurate?)
- Deterministic models are simpler and faster
- Stochastic may be preferred for consistency if already using stochastic for other things

Approaches for Determining FX Volatility

1. **Historical Approach**
   - $\sigma$ = standard deviation of FX return over some period of time
   - Longer-term view, less focus on current market
   - Use *N* years of data for an *N*-year block
   - Use credible, relevant data
2. **Market Volatility Approach**
   - $\sigma$ = implied volatility in market prices
   - Shorter-term, market-consistent focus (risk-neutral)
   - Calibrate volatilities to market prices
   - Volatility term structure won't match historical

Validating FX Models

# Source: QFIA-124 (Part II)

**If you shock $r_D$ upward +X bps:**

- Domestic equity and bond returns should increase +X bps
- Foreign equity and bond (domestic) returns should increase +X bps
- Domestic bonds will immediately fall in value

**If you shock $r_F$ upward +X bps:**

- No effect on domestic equities or bonds
- Foreign bonds fall in value in their own currency
- No net effect on domestic return of foreign assets

Equity Scenarios

## Source: QFIA-124 (Part II)

Arbitrage-free, risk-neutral equity scenario rates:

$$S_t = S_{t-1} e^{\left(F_t(t+dt)-q-\frac{\sigma_t^2}{2}\right)dt+\sigma_t\phi_t}$$

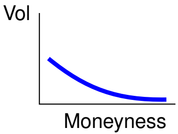
- $F_t$ = forward rate, $q$ = dividend rate, $\phi_t$ = random draw
- Derived from geometric Brownian motion and Black-Scholes
- Key advantage: no negative rates
- Disadvantage: volatility assumption is too simple
  - Could make $\sigma_t$ vary with security price
  - Could make $\sigma_t$ a stochastic process
- Must calibrate risk-neutral $\sigma$ assumption
  - Deterministic: Bootstrap forward volatilities from implied
  - Stochastic: Fit to market using least squares
- Convert to realistic rates by adding a risk premium to $F_t$

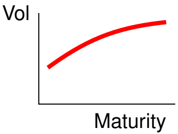
Stylized Facts about Equity Returns

# Source: QFIA-124 (Part II)

These are all problems with the basic Black-Scholes approach...

1. Implied volatility changes with market conditions
    - Varies with in-the-moneyness and maturity




    - Call and put volatility is not symmetric
2. Equity volatility falls as equity values rise
3. Actual returns have fatter tail than normal distribution
4. Volatility has a long memory
5. Volatility can cluster for a short period

Describe structural models.

# Source: QFIA-124 (Part III)

- Structural Models - use public information to estimate distribution of firm's asset value
  - $D = Pr(A < T)$
  - $A$ is the random variable for the asset value of the firm
  - $D$ is the cumulative default rate
  - $T$ is the default threshold
- Merton Model (1974) is a classic example
- Risk Neutral Probability of Default = $N(-d_2)$

What is the risk-neutral probability of default using a structural model?

# Source: QFIA-124 (Part III)

- Risk Neutral Probability of Default = $N(-d_2)$

State three disadvantages of structural models.

Source: QFIA-124 (Part III)

1. Complex
2. Difficult to calibrate
3. May not have realistic short-term spreads

Describe reduced-form models.

## Source: QFIA-124 (Part III)

- Uses **hazard rates** – forward probability of default
- Use public information such as credit default swaps to create an assumption for the hazard rate
- Recent models have been more focused on modeling hazard rates through a stochastic process

State three alternatives to stochastic models.





Implementation Steps for Stochastic Models





Risk-Neutral Scenarios

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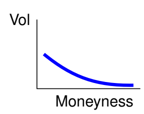

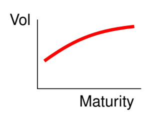


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