QFII-104-14 Correlation Pitfalls And Alternatives

Elliptical Distributions

Linear correlation is unproblematic when dealing with **elliptical distributions**, which include:

- Multivariate normal distribution
- Multivariate t-distribution

Fallacies of Correlations

1. **Marginal distributions and correlation determine the joint distribution**
   - Outside the elliptical world, there are infinitely many multivariate distributions that could fit
   - Correlation does not tell us anything about the degree of dependence in the tails of the underlying distribution
2. **Given marginal distributions F_1 and F_2 for X_1 and X_2, all linear correlations between -1 and 1 can be attained through a suitable specification of the joint distribution F**
   - In general, the attainable correlations depend on F_1 and F_2, and form a closed interval $[\rho_{min}, \rho_{max}]$ (containing zero) that is a subset of $[-1, 1]$

Problems of Correlation as a Dependency Measure

1. Correlation is simply a scalar measure of dependency
2. Possible values of correlation depend on the marginal distributions of risks (i.e. not all values between -1 and 1 are attainable)
3. Perfectly positively dependent risks do not necessarily have a correlation of 1, and vice versa
4. A correlation of zero does not indicate independence of risks
5. Correlation is not invariant under transformation of risks
   - **Example**: $\ln(X_1)$ and $\ln(X_2)$ do not have the same correlation as X_1 and X_2
6. Correlation is only defined when the variances of the risks are finite
   - Not appropriate for very heavy-tailed risks where variances appear infinite

Alternatives to Linear Correlation

1. **Spearman's rank correlation**

$$\rho_S(X_i, X_j) = \rho\left(F_i(X_i), F_j(X_j)\right)$$

Disadvantages:

- Still has deficiencies 1 and 4 listed in previous slide
- Cannot be manipulated as easily as linear correlation
- Knowledge of rank correlation still will not allow us to find a *unique* multivariate distribution

2. **Copulas**

- Represent a way of extracting the dependence structure from the joint distribution
- By simply applying a copula function to prescribed marginal distributions, we can construct a multivariate distribution
- Best way to model dependency between risks! ✓