The Infinite Actuary Exam STAM Online Course
A.1.1. Describing Distribuitons
Last updated April 11, 2018

1. [3-CAS.F03.17] Losses have an Inverse Exponential distribution. The mode is 10,000. Calculate the median.

   A. Less than 10,000
   B. At least 10,000, but less than 15,000
   C. At least 15,000, but less than 20,000
   D. At least 20,000, but less than 25,000
   E. At least 25,000

From the tables, the mode is $\theta/2 = 10{,}000$, so $\theta = 20{,}000$.
To find the median, set $0.5 = F(x) = e^{-20{,}000/x}$ so $x = -20{,}000/\ln(0.5) = \boxed{28{,}854}$

Alternatively, the median is $\text{VaR}_{0.5}(X) = \theta(-\ln 0.5)^{-1} = 28{,}854$

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2. [3-CAS.F03.19] For a loss distribution where $x \geq 2$, you are given:

   (i) The hazard rate function: $h(x) = \dfrac{z^2}{2x}$, for $x \geq 2$
   (ii) A value of the distribution function: $F(5) = 0.84$.

   Calculate z.

   A. 2 B. 3 C. 4 D. 5 E. 6

$$H(5) = \int_0^5 h(x)dx = \int_0^2 0\,dx + \int_2^5 \frac{z^2}{2x}dx = \frac{z^2}{2}\ln(5) - \frac{z^2}{2}\ln(2) = \frac{z^2}{2}\ln(2.5)$$

$F(5) = 0.84 \Rightarrow S(5) = 1 - 0.84 = 0.16 = e^{-H(5)}$ and $H(5) = -\ln(0.16) = \dfrac{z^2}{2}\ln(2.5)$ so $\boxed{z = 2}$

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3. The density of Y is proportional to y^2 for $0 < y < 3$, and is 0 otherwise. Find the 80th percentile of Y.

   A. 0.9 B. 1.3 C. 1.8 D. 2.3 E. 2.8

$f(y) = cy^2, 0 < y < 3$, so $1 = \displaystyle\int_0^3 cy^2dy = c\,\frac{y^3}{3}\Big|_0^3 = 9c.$

That gives us $c = 1/9$, and for the 80th percentile, $0.8 = \displaystyle\int_0^t \frac{1}{9}y^2dy = \frac{t^3}{27}$ so $\boxed{t = 2.78}$

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4. A nonnegative random variable has a hazard rate function of $h(x) = A + e^{2x}, x \geq 0$. You are also given $S(0.4) = 0.5$.

   Determine the value of A.

   A. Less than 0.5
   B. At least 0.5, but less than 1.0
   C. At least 1.0, but less than 1.5
   D. At least 1.5, but less than 2.0
   E. At least 2.0

$$\begin{aligned} S(t) &= e^{-H(t)} \\ -\ln S(t) = H(t) &= \int_0^{0.4} A + e^{2x} dx \\ -\ln(0.5) &= 0.4A + \frac{1}{2}\left(e^{0.8} - 1\right) \\ A &= \boxed{0.2} \end{aligned}$$