

The Infinite Actuary Exam STAM Online Course

A.1.4. Joint Distributions

Last updated April 11, 2018

1. X and Y are discrete variables whose joint distribution $P[X = x, Y = y] = p(x, y)$ is given by

$$\begin{aligned} p(1, 1) &= 0.12 & p(2, 1) &= 0.06 & p(3, 1) &= 0.12 \\ p(1, 2) &= 0.00 & p(2, 2) &= 0.12 & p(3, 2) &= 0.08 \\ p(1, 3) &= 0.20 & p(2, 3) &= 0.05 & p(3, 3) &= 0.15 \\ p(1, 4) &= 0.05 & p(2, 4) &= 0.02 & p(3, 4) &= 0.03 \end{aligned}$$

Find $P[X > Y]$

A. 0.26

B. 0.30

C. 0.32

D. 0.35

E. 0.39

There are 3 cases in which $X > Y$, namely $(2, 1)$, $(3, 1)$ and $(3, 2)$. Summing their probabilities gives $0.06 + 0.12 + 0.08 = \boxed{0.26}$

2. X and Y are discrete variables whose joint distribution $P[X = x, Y = y] = p(x, y)$ is given by

$$\begin{aligned} p(0, 1) &= 0.2 & p(1, 1) &= 0.1 \\ p(1, 2) &= 0.3 & p(2, 2) &= 0.1 \\ p(2, 3) &= 0.1 & p(3, 3) &= 0.2 \end{aligned}$$

Find the coefficient of variation of Y .

A. 0.30

B. 0.39

C. 0.60

D. 2.58

E. 3.33

$$P[Y = 1] = 0.3 \quad P[Y = 2] = 0.4 \quad P[Y = 3] = 0.3$$

$$E[Y] = 0.3 \cdot 1 + 0.4 \cdot 2 + 0.3 \cdot 3 = 2$$

$$\text{Var}[Y] = 0.3 \cdot (1 - 2)^2 + 0.4 \cdot (2 - 2)^2 + 0.3 \cdot (3 - 2)^2 = 0.6$$

$$\text{CV}[Y] = \frac{\sqrt{0.6}}{2} = \boxed{0.387}$$
