

The Infinite Actuary Exam STAM Online Course

A.1.5. Conditional Moments

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1. X and Y are discrete variables whose joint distribution $P[X = x, Y = y] = p(x, y)$ is given by

$$\begin{aligned} p(0, 1) &= 0.2 & p(1, 1) &= 0.1 \\ p(1, 2) &= 0.3 & p(2, 2) &= 0.1 \\ p(2, 3) &= 0.1 & p(3, 3) &= 0.2 \end{aligned}$$

Find the coefficient of variation of Y conditioned on X being positive.

- A. 0.29 B. 0.31 C. 0.33 D. 0.35 E. 0.37
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$$\begin{aligned} P[X > 0] &= 1 - 0.2 = 0.8 \\ P[Y = 1 \mid X > 0] &= \frac{0.1}{0.8} \\ P[Y = 2 \mid X > 0] &= \frac{0.4}{0.8} \\ P[Y = 3 \mid X > 0] &= \frac{0.3}{0.8} \\ E[Y \mid X > 0] &= 2.25 \\ \text{Var}[Y \mid X > 0] &= \frac{7}{16} \\ \text{CV}[Y \mid X > 0] &= \frac{\sqrt{7/16}}{2.25} = \boxed{0.294} \end{aligned}$$

2. Let N be the value rolled by a fair six-sided die. Suppose that I then flip N independent fair coins. What is the expected number of heads?

- A. 1/4 B. 3/4 C. 3/2 D. 7/4 E. 7/2
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If N had a fixed, known value it would be easy, so we want to use double expectation. $E[H] = E[E[H \mid N]] = E[N/2] = (7/2)/2 = \boxed{7/4}$

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3. (Based on [110.F86.41]) Let $E[X \mid Y = y] = 3y$, $\text{Var}[X \mid Y = y] = 2$ and let Y be an exponential random variable with mean 1.

What is $\text{Var}[X]$?

- A. 3 B. 5 C. 9 D. 11 E. 20
-

Using the conditional variance formula,

$$\text{Var}(X) = E[\text{Var}(X \mid Y = y)] + \text{Var}[E(X \mid Y = y)]$$

$$\begin{aligned}
&= E[2] + \text{Var}[3Y] = 2 + 3^2 \text{Var}(Y) \\
&= 2 + 9 \cdot 1^2 = \boxed{11}
\end{aligned}$$

4. [3.Sample.10] An insurance company is negotiating to settle a liability claim. If a settlement is not reached, the claim will be decided in the courts 3 years from now. You are given:

- There is a 50% probability that the courts will require the insurance company to make a payment. The amount of the payment, if there is one, has a lognormal distribution with mean 10 and standard deviation 20.
- In either case, if the claim is not settled now, the insurance company will have to be pay 5 in legal expenses, which will be paid when the claim is decided 3 years from now.
- The most that the insurance company is willing to pay to settle the claim is the expected present value of the claim and legal expenses, plus 0.02 times the variance of the present value.
- Present values are calculated using $i = 0.04$.

Calculate the insurance company's maximum settlement value for this claim.

A. 8.89 B. 9.93 C. 12.45 D. 12.89 E. 13.53

Let X be the present value of the claim amount if no settlement is reached. Then

$$\begin{aligned}
P[X = 0] &= 0.5 \\
E[X \mid X > 0] &= 1.04^{-3} \cdot 10 \\
\text{Var}[X \mid X > 0] &= (1.04^{-3} \cdot 20)^2 \\
\text{Var}[X] &= E[\text{Var}[X \mid \text{Is } X > 0?]] + \text{Var}[E[X \mid \text{Is } X > 0?]] \\
&= 0.5 \cdot 1.04^{-6} \cdot 20^2 + 0.5^2 \cdot (1.04^{-3} \cdot 10)^2 = 177.82 \\
\text{Max Settlement} &= 0.5 \cdot 1.04^{-3} \cdot 10 + 5 \cdot 1.04^{-3} + 0.02 \cdot 177.82 = \boxed{12.45}
\end{aligned}$$