The Infinite Actuary Exam STAM Online Course
A.1.6. Mixtures
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1. An insurer groups its policyholders into two groups. Losses from members of group A have a Burr distribution with $\theta = 10$, $\gamma = 3$ and $\alpha = 1$, while losses from members of group B have a Burr distribution with $\theta = 10, \gamma = 3$ and $\alpha = 2$.

   If 40% of losses are from policyholders from group A, find the median loss amount

   A. 8.07 B. 8.17 C. 8.27 D. 8.37 E. 8.47

$$F(x) = 0.4 \cdot \left[1 - \frac{1}{1+(x/10)^3}\right] + 0.6 \cdot \left[1 - \left(\frac{1}{1+(x/10)^3}\right)^2\right]$$

$$0.5 = 1 - 0.4u - 0.6u^2 \quad \text{where } u = \frac{1}{1+(x/10)^3}$$

$$0.6u^2 + 0.4u - 0.5 = 0$$

$$u = \frac{1}{1+(x/10)^3} = 0.638$$

$$1 + \left(\frac{x}{10}\right)^3 = 1.566$$

$$x = \boxed{8.27}$$

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2. An insurance company has 3 types of customers, whose annual losses are described below. Find the variance of the annual loss of a randomly chosen customer.

Type	Proportion of Customers	Average Annual Loss	Variance of Annual Losses
Low	50%	10	20
Med	30%	20	50
High	20%	40	100

   A. 45 B. 80 C. 123 D. 174 E. 219

$$\begin{aligned}
\mathrm{Var}[X] &= \mathrm{E}[\mathrm{Var}[X \mid \text{case}]] + \mathrm{Var}[\mathrm{E}[X \mid \text{case}]] \\
\mathrm{E}[\mathrm{Var}[X \mid \text{case}]] &= 0.5 \cdot 20 + 0.3 \cdot 50 + 0.2 \cdot 100 = 45 \\
\mathrm{Var}[\mathrm{E}[X \mid \text{case}]] &= 0.5 \cdot 10^2 + 0.3 \cdot 20^2 + 0.2 \cdot 40^2 - (0.5 \cdot 10 + 0.3 \cdot 20 + 0.2 \cdot 40)^2 \\
&= 129 \\
\mathrm{Var}[X] &= 45 + 129 = \boxed{174}
\end{aligned}$$

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3. An insurance company has two types of customers: high risk customers, whose loss amounts have mean 20 and variance 50, and low risk customers, whose losses have mean 10. The expected amount of a randomly chosen loss is 14, and the variance of a randomly selected loss is 62. Find the variance of a loss from a low risk customer.

A. 12 B. 18 C. 30 D. 48 E. 70

Let p denote the proportion of low risk customers.

$$\begin{aligned}
14 &= 10p + 20(1-p) \\
p &= 0.6 \\
\mathrm{Var}[X] &= \mathrm{E}[\mathrm{Var}[X \mid \mathrm{Class}]] + \mathrm{Var}[\mathrm{E}[X \mid \mathrm{Class}]] \\
62 &= 0.6 \cdot \mathrm{Var}[X \mid \mathrm{Low}] + 0.4 \cdot 50 + 0.4 \cdot 0.6 \cdot (20-10)^2 \\
\mathrm{Var}[X \mid \mathrm{Low}] &= \boxed{30}
\end{aligned}$$

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4. [3-CAS.F04.29] High-Roller Insurance Company insures the cost of injuries to the employees of ACME Dynamic Manufacturing, Inc.

- 30% of injuries are "Fatal" and the rest are "Permanent Total" (PT). There are no other injury types.
- Fatal injuries follow a log-logistic distribution with $\theta = 400$ and $\gamma = 2$.
- PT injuries for a log-logistic distribution with $\theta = 600$ and $\gamma = 2$.
- There is a \$750 deductible per injury.

Calculate the probability that an injury will result in a claim to High-Roller.

A. Less than 30%
B. At least 30%, but less than 35%
C. At least 35%, but less than 40%
D. At least 40%, but less than 45%
E. At least 45%

Let X be a random injury amount.

$$\begin{aligned}
\mathrm{P}[X > 750] &= 0.3 \cdot \mathrm{P}[X > 750 \mid \mathrm{Fatal}] + 0.7 \cdot \mathrm{P}[X > 750 \mid \mathrm{PT}] \\
&= 0.3 \cdot \left[1 - \frac{(750/400)^2}{1+(750/400)^2}\right] + 0.7 \cdot \left[1 - \frac{(750/600)^2}{1+(750/600)^2}\right] \\
&= \boxed{0.3396}
\end{aligned}$$

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5. The number of annual claims filed by low risk individuals is Poisson with mean 2, while the number of annual claims filed by high risk individuals is Poisson with mean 3. Suppose that 40% of individuals are low risk and 60% are high risk. Let W be the number of annual claims filed by a randomly chosen individual. Find $\mathrm{Var}[W] - \mathrm{E}[W]$.

A. -1.2 B. -0.24 C. 0 D. 0.24 E. 1.2

Here we have a mixture, not a sum.

$$\begin{aligned}
\mathrm{E}[W] &= \mathrm{E}[\mathrm{E}[W \mid \text{Case}]] = 0.4 \cdot 2 + 0.6 \cdot 3 = 2.6 \\
\mathrm{Var}[W] &= \mathrm{E}[\mathrm{Var}[W \mid \text{Case}]] + \mathrm{Var}[\mathrm{E}[W \mid \text{Case}]] \\
&= 0.4 \cdot 2 + 0.6 \cdot 3 + (3-2)^2 \cdot 0.4 \cdot 0.6 = 2.84 \\
\mathrm{Var}[W] - \mathrm{E}[W] &= 2.84 - 2.6 = \boxed{0.24}
\end{aligned}$$

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