The Infinite Actuary Exam STAM Online Course
A.2.3. Practice Problems on Pareto Distributions

1. [3.F01.37] For watches produced by a certain manufacturer:

   (i) Lifetimes follow a single-parameter Pareto distribution with $\alpha > 1$ and $\theta = 4$.
   (ii) The expected lifetime of a watch is 8 years.

   Calculate the probability that the lifetime of a watch is at least 6 years.

   A. 0.44 B. 0.50 C. 0.56 D. 0.61 E. 0.67

2. [3.S06.25] Calculate the skewness of a Pareto distribution with $\alpha = 4$ and $\theta = 1{,}000$.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

3. Losses are a 2-point mixture. 30% of the time, losses come from a Pareto distribution with $\alpha = 3$ and $\theta = 10$, and 70% of the time losses come from a Pareto distribution with $\alpha = 6$ and $\theta = 10$. What is the median loss amount?

A. 1.50 B. 1.53 C. 1.57 D. 1.60 E. 1.64

4. [M.S05.9] A loss, X, follows a 2-parameter Pareto distribution with $\alpha = 2$ and unspecified parameter θ. You are given:

$$\mathrm{E}\left[X - 100 \mid X > 100\right] = \frac{5}{3}\mathrm{E}\left[X - 50 \mid X > 50\right]$$

Calculate $\mathrm{E}\left[X - 150 \mid X > 150\right]$.

A. 150 B. 175 C. 200 D. 225 E. 250

5. [M.S05.34] The distribution of a loss, X , is a two-point mixture:

(i) With probability 0.8, X has a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 100$.

(ii) With probability 0.2, X has a two-parameter Pareto distribution with $\alpha = 4$ and $\theta = 3000$.

Calculate $\Pr(X \leq 200)$.

A. 0.76 B. 0.79 C. 0.82 D. 0.85 E. 0.88

Suppose that X is a mixture of two distributions, one of which is a single parameter Pareto distribution with $\alpha = 3$ and $\theta = 100$ and the other is a single parameter Pareto with $\alpha = 3$ and $\theta = 100 + \delta$ for some value of δ.

If $\mathrm{E}[X] = 165$ and $\mathrm{E}\left(X^2\right) = 37{,}200$, then what is δ? 1020304050 Let p be the probability that we are selecting from the distribution with $\theta = 100 + \delta$. Then

$$\begin{aligned}
\mathrm{E}[X] &= (1-p)\frac{3 \cdot 100}{3-1} + p\frac{3(100+\delta)}{3-1} \\
165 &= \frac{3}{2}(100 - 100p + 100p + \delta p) = 150 + 1.5\delta p \\
10 &= \delta p
\end{aligned}$$

and for the second moment,

$$\begin{aligned}
\mathrm{E}[X^2] = 37{,}200 &= (1-p)\frac{3 \cdot 100^2}{3-2} + p\frac{3(100+\delta)^2}{3-2} \\
12{,}400 &= (1-p)100^2 + p \cdot 100^2 + 2p \cdot 100\delta + p\delta^2 \\
2{,}400 &= 200(10) + 10(\delta) \qquad \text{by plugging in } 10 = p\delta \\
\delta &= \boxed{40}
\end{aligned}$$

6. Based on [4B.F99.19] Suppose that X has a Pareto distribution with $\alpha = 2$ and $\theta = 10,000$, and Y has a Burr distribution with parameters $\alpha = 1, \gamma = 2$ and $\theta = \sqrt{20,000}$.

   Let r be the ratio of $\mathrm{P}[X > d]$ to $\mathrm{P}[Y > d]$. What is $\lim\limits_{d\to\infty} r$?

   A. 0 B. 0.25 C. 1 D. 5,000 E. ∞