A.2 Key Distributions - Outline

Continuous Uniform Variables

Suppose that X is uniform on (a, b).

Then all points in the interval are "equally likely" so for $a < x < b$,

$$f(x) = \frac{1}{\text{length of interval}} = \frac{1}{b-a}$$

$$F(x) = \frac{\text{amount of interval} \leq x}{\text{length of interval}} = \frac{x-a}{b-a}$$

$$\mathrm{E}[X] = \frac{a+b}{2}$$

$$\mathrm{Var}[X] = \frac{(b-a)^2}{12}$$

Not included in distribution table

Example

For low risk insureds, losses are uniform on $(0, 10)$. For high risk insureds, losses are uniform on $(4, 20)$. If 30% of insureds are low risk, what is the survival function for a randomly selected loss amount?

$$\mathrm{P}[X > x] = 0.3 \cdot \mathrm{P}[X > x \mid \text{low}] + 0.7 \cdot \mathrm{P}[X > x \mid \text{high}]$$
$$= \begin{cases} 0.3 \cdot \dfrac{10-x}{10} + 0.7 \cdot 1 & 0 < x < 4 \\ 0.3 \cdot \dfrac{10-x}{10} + 0.7 \cdot \dfrac{20-x}{16} & 4 < x < 10 \\ 0.3 \cdot 0 + 0.7 \cdot \dfrac{20-x}{16} & 10 < x < 20 \end{cases}$$

Formula comparison

For $\alpha \geq 1$ an integer,

	Exponential	Gamma	Pareto	Uniform(a, b)
$f(x)$	$\frac{1}{\theta} e^{-x/\theta}$	$\frac{x^{\alpha-1}}{\theta^\alpha (\alpha-1)!} e^{-x/\theta}$	$\frac{\alpha\theta^\alpha}{(\theta+x)^{\alpha+1}}$	$\frac{1}{b-a}$
$F(x)$	$1 - e^{-x/\theta}$	$1 - \sum_{i=0}^{\alpha-1} \frac{(x/\theta)^i}{i!} e^{-x/\theta}$	$1 - \left(\frac{\theta}{x+\theta}\right)^\alpha$	$\frac{x-a}{b-a}$
$S(x)$	$e^{-x/\theta}$	$\sum_{i=0}^{\alpha-1} \frac{(x/\theta)^i}{i!} e^{-x/\theta}$	$\left(\frac{\theta}{x+\theta}\right)^\alpha$	$\frac{b-x}{b-a}$
$\mathrm{E}[X]$	θ	$\alpha\theta$	$\frac{\theta}{\alpha-1}$	$\frac{a+b}{2}$
$\mathrm{Var}[X]$	θ^2	$\alpha\theta^2$	$\frac{\theta^2\alpha}{(\alpha-1)^2(\alpha-2)}$	$\frac{(b-a)^2}{12}$
$e_X(d)$	θ	messy	$\frac{\theta+d}{\alpha-1}$	$\frac{b-d}{2}$

where $e_X(d) = \mathrm{E}[X - d \mid X > d]$

Exercise 1

Low risk individuals have losses that are uniformly distributed on $(0, 50)$, while high risk individuals have losses that are uniform on $(0, 100)$. The expected loss size of a randomly chosen individual is 35. What is the variance of the loss amount of a randomly chosen individual?

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Let $p = \mathrm{P}[\text{low}]$. Then

$$\begin{aligned}
\mathrm{E}[X] &= p \cdot \mathrm{E}[X \mid \text{low}] + (1-p) \cdot \mathrm{E}[X \mid \text{high}] \\
35 &= p \cdot 25 + (1-p) \cdot 50 \\
p &= 0.6 \\
\mathrm{Var}[X] &= \mathrm{E}[\mathrm{Var}[X \mid \text{Case}]] + \mathrm{Var}[\mathrm{E}[X \mid \text{Case}]] \\
&= 0.6 \cdot \frac{50^2}{12} + 0.4 \cdot \frac{100^2}{12} + (25-50)^2 \cdot 0.6 \cdot 0.4 \\
&= \boxed{608.33}
\end{aligned}$$

Exercise 2

Suppose that X is an exponential random variable with mean μ, where μ is a Pareto random variable with parameters $\alpha = 3$ and $\theta = 100$. Find the variance of X.

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$$\begin{aligned}
\mathrm{Var}[X] &= \mathrm{E}[\mathrm{Var}(X \mid \mu)] + \mathrm{Var}[\mathrm{E}(X \mid \mu)] \\
&= \mathrm{E}[\mu^2] + \mathrm{Var}[\mu] \\
&= \frac{2 \cdot 100^2}{(3-1)(3-2)} + \left(\frac{100}{3-1}\right)^2 \frac{3}{3-2} \\
&= \boxed{17{,}500}
\end{aligned}$$