The Infinite Actuary Exam STAM Online Course
A.3.1. Scale Parameters
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1. Losses in 2009 have a Gamma distribution with mean 200 and variance $20,000$. If inflation is 10%, what is the variance of a loss in 2010?

   A. 20,000 B. 22,000 C. 24,200 D. 26,620 E. 28,000

The short answer is $\text{Var}(1.1X) = 1.1^2\text{Var}(X)$, giving us $1.21 \cdot 20{,}000 = \boxed{24{,}200}$

The longer approach is that $\text{E}[X] = \alpha\theta = 200$ and $\text{Var}(X) = \alpha\theta^2 = 20{,}000$ so $\theta = 100$ and $\alpha = 2$. With 10% inflation, the new $\theta' = 1.1 \cdot 100 = 110$ and the new variance is $\alpha(\theta')^2 = 2 \cdot 110^2 = 24{,}200$.

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2. [4B.S90.37] Liability claim severity follows a Pareto distribution with a mean of 25,000 and parameter $\alpha = 3$. If inflation increases all claims by 20%, the probability of a claim exceeding 100,000 increases by what amount?

   A. Less than 0.02
   B. At least 0.02, but less than 0.03
   C. At least 0.03, but less than 0.04
   D. At least 0.04, but less than 0.05
   E. At least 0.05

For a Pareto, $\text{E}[X] = \dfrac{\theta}{\alpha - 1} = \dfrac{\theta}{2}$. Setting equal to 25,000 gives us $50{,}000 = \theta$. After inflation, the new theta is $1.2 \cdot 50{,}000 = 60{,}000$, so

$$\begin{aligned}
\text{P}[X > 100{,}000] \left(\frac{\theta}{\theta + 100{,}000}\right)^{\alpha} &= \left(\frac{1}{3}\right)^3 = 0.037 \text{ before inflation} \\
\text{P}[X > 100{,}000] \left(\frac{60{,}000}{60{,}000 + 100{,}000}\right)^{\alpha} &= \left(\frac{6}{16}\right)^3 = 0.052 \text{ after inflation}
\end{aligned}$$

for an increase of $\boxed{0.0157}$

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3. [3-CAS.S04.34] Claim severities are modeled using a continuous distribution and inflation impacts claims uniformly at an annual rate of i. Which of the following are true statements regarding the distribution of claim severities after the effect of inflation?

(i) An Exponential distribution will have scale parameter $(1+i)\theta$

(ii) A 2-parameter Pareto distribution will have scale parameter $(1+i)\alpha$ and $(1+i)\theta$.

(iii) A Paralogistic distribution will have scale parameter $\theta/(1+i)$.

A. (i) only B. (iii) only C. (i) and (ii) only D. (ii) and (iii) only E. (i), (ii), and (iii)

Only statement (i) is true so the answer is $\boxed{A}$. (ii) is false since α is not a scale parameter and shouldn't change, and (iii) is false because we should still get $(1+i)\theta$ as in (i).

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4. [4B.F95.06] You are given the following:

(i) In 1994, losses follow a Pareto distribution with parameters $\theta = 500$ and $\alpha = 1.5$.

(ii) Inflation of 5% affects all losses uniformly from 1994 to 1995.

What is the median of the portion of the 1995 loss distribution above 200?

A. Less than 600
B. At least 600, but less than 620
C. At least 620, but less than 640
D. At least 640, but less than 660
E. At least 660

In 2009, our new θ is $1.05 \cdot 500 = 525$. We want the median of the portion of the loss above 200, so $0.5 = \mathrm{P}[X \le x \mid X > 200]$. It is often easier to work with survival functions when conditioning on being above something, which gives us $1 - 0.5 = \mathrm{P}[X > x \mid X > 200] = S(x)/S(200)$. Plugging in $S(x)$ gives

$$\begin{aligned}
1 - 0.5 = 0.5 &= \frac{\left(\dfrac{525}{x+525}\right)^{1.5}}{\left(\dfrac{525}{200+525}\right)^{1.5}} \\
(x+525)^{1.5} &= 2(725)^{1.5} \\
x &= \boxed{625.86}
\end{aligned}$$

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5. [1.Sample.17] An actuary is reviewing a study she performed on the size of claims made ten years ago under homeowners insurance policies. In her study, she concluded that the size of claims followed an exponential distribution and that the probability that a claim would be less than \$1,000 was 0.250.

   The actuary feels that the conclusions she reached in her study are still valid today with one exception: every claim made today would be twice the size of a similar claim made ten years ago as a result of inflation.

   Calculate the probability that the size of a claim made today is less than \$1,000.

   A. 0.063 B. 0.125 C. 0.134 D. 0.163 E. 0.250

$0.25 = F(1{,}000) = 1 - e^{-1000/\theta}$ so $\theta = 3{,}476$. Claim sizes double, giving us a new θ of 6,952. The new $F(1000)$ is $1 - e^{-1000/6952} = \boxed{0.134}$

Alternatively, $\theta' = 2\theta$, so the new survival function at 1,000 is $e^{-1000/(2\theta)} = \left(e^{-1000/\theta}\right)^{1/2} = 0.75^{1/2}$ and the new CDF at 1,000 is $1 - 0.75^{1/2} = 0.134$.

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6. Losses in 2012 had density $4x^{-5}$ for $x > 1$ where x denotes the loss amount in millions. Inflation of 6% affects all claims uniformly from 2012 to 2013. Find the probability that losses in 2013 exceed 1.8 million.

   A. 0.09 B. 0.10 C. 0.11 D. 0.12 E. 0.13

$$\begin{aligned}
\mathrm{P}[X > t] &= \int_t^{\infty} 4x^{-5}dx = \left.\frac{-1}{x^4}\right|_t^{\infty} = \frac{1}{t^4} \\
\mathrm{P}[1.06X > 1.8] &= \mathrm{P}\left[X > \frac{1.8}{1.06}\right] \\
&= \frac{1}{(1.8/1.06)^4} = \boxed{0.12}
\end{aligned}$$

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7. [4B.S99.17] You are given the following:

- Claims have a Pareto distribution with parameters θ (unknown) and $\alpha = 2$.
- Inflation of 6% affects all claims uniformly from 1998 to 1999.
- r is the ratio of the proportion of claims that will exceed d in 1999 year to the proportion of claims that exceed d in 1998.

Determine the limit of r as d goes to infinity.

A. Less than 1.05
B. At least 1.05, but less than 1.10
C. At least 1.10, but less than 1.15
D. At least 1.15, but less than 1.20
E. At least 1.20

Since we have 6% inflation, the new parameter θ' for next year is 1.06θ. So

$$\begin{aligned} r &= \frac{S_{\text{next year}}(d)}{S_{\text{this year}}(d)} = \frac{\left(\frac{1.06\theta}{1.06\theta+d}\right)^2}{\left(\frac{\theta}{\theta+d}\right)^2} \\ &= \frac{1.06^2\theta^2}{(1.06\theta+d)^2} \cdot \frac{(\theta+d)^2}{\theta^2} \\ &= 1.06^2\left(\frac{\theta+d}{1.06\theta+d}\right)^2 \\ \lim_{d\to\infty} r &= 1.06^2\left(\frac{\theta/d+1}{1.06\theta/d+1}\right)^2 = 1.06^2 = \boxed{1.1236} \end{aligned}$$

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8. Losses have a lognormal distribution with $\mu = 6.9078$ and $\sigma = 1.5174$.

Determine the percentage increase in the number of losses that exceed 1,000 if all losses are increased by 10%.

A. Less than 2%
B. At least 2%, but less than 4%
C. At least 4%, but less than 6%
D. At least 6%, but less than 8%
E. At least 8%

If $X \sim \text{LN}(6.9078, 1.5174^2)$ then $Y = \ln(X) \sim \text{N}(6.9078, 1.5174^2)$.

$$\begin{aligned} \text{P}[X > 1{,}000] &= \text{P}[Y > \ln(1{,}000)] = 1 - \Phi\left(\frac{\ln(1,000) - 6.9078}{1.5174}\right) = 1 - \Phi(0) = 0.5 \\ \text{P}[1.1X > 1{,}000] &= \text{P}[X > 1{,}000/1.1] \end{aligned}$$

$$= \mathrm{P}[Y > \ln(1{,}000) - \ln(1.1)] = 1 - \Phi\left(\frac{\ln(1,000) - \ln(1.1) - 6.9078}{1.5174}\right)$$
$$= 1 - \Phi(-0.06) = 0.5239$$

for a $0.5239/0.5 - 1 = 4.78\%$ increase.

If you want to adjust the parameters for the inflation, σ remains the same and μ increases by $\ln(1.1)$.

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9. Loss amounts in 2011 have a lognormal distribution with parameters $\mu = -0.8$ and $\sigma^2 = 1.69$. If there is annual inflation of 5% per year, what is the variance of loss amounts in 2014?

A. 2.0 B. 3.2 C. 4.4 D. 5.6 E. 6.5

Let X be a loss amount in 2011, and Y a loss in 2014. Since X and Y are lognormal, we ultimately want to take natural logs to understand the distribution. $\mathrm{P}[\ln(Y) \le \ln(y)] = \mathrm{P}[Y \le y] = \mathrm{P}[(1.05)^3 X \le y] = \mathrm{P}[3\ln(1.05) + \ln X \le \ln(y)]$. Since $\ln(X)$ is a normal with $\mu = -0.8$ and $\sigma^2 = 1.69$, this means that $\ln(Y)$ is a normal with $\mu' = -0.8 + 3\ln(1.05) = -0.6536$ and $\sigma^2 = 1.69$.

More generally, if we have n years of inflation at a rate of i, then for a lognormal, $\mu' = mu + n\ln(1+i)$, and σ^2 doesn't change.

For a lognormal, the variance is

$$\mathrm{Var}(Y) = e^{2\mu' + 2\sigma^2} - \left(e^{\mu' + \sigma^2/2}\right)^2 = \boxed{6.48}$$

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10. Loss amounts in 2010 have a lognormal distribution with mean 10 and variance 40. Loss amounts increase due to inflation by 5% each year. What is the probability that a loss in 2012 will exceed 20?

A. 0.03 B. 0.05 C. 0.07 D. 0.09 E. 0.11

Let X be a loss in 2010. Then

$$\begin{aligned}
\mathrm{E}[X] &= e^{\mu + \sigma^2/2} = 10 \\
\mathrm{E}[X^2] &= e^{2\mu + 2\sigma^2} = 40 + 10^2 = 140 \\
2\mu + \sigma^2 &= 2\ln(10) \\
2\mu + 2\sigma^2 &= \ln(140) \\
\sigma^2 &\approx 0.3365 \qquad \mu \approx 2.1343 \\
\mathrm{P}[1.05^2 X > 20] &= \mathrm{P}[X > \ln(20) - 2\ln(1.05)] \\
&= 1 - \Phi\left(\frac{\ln(20) - 2\ln(1.05) - \mu}{\sigma}\right) \\
&= 1 - \Phi(1.32) = \boxed{0.0934}
\end{aligned}$$

Alternatively, we could have found μ and σ^2 as above, then said that losses in 2012 were lognormal($\mu' = \mu + 2\ln(1.05), \sigma'^2 = \sigma^2$)