

Last updated August 24, 2018.

1. [First Pass; 3.F01.37] For watches produced by a certain manufacturer:

- Lifetimes follow a single-parameter Pareto distribution with  $\alpha > 1$  and  $\theta = 4$ .
- The expected lifetime of a watch is 8 years.

You are given the following information about a single-parameter Pareto:

- $f(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta$
- $F(x) = 1 - \left(\frac{\theta}{x}\right)^\alpha, x > \theta$
- $E[X] = \frac{\alpha\theta}{\alpha-1}$

Calculate the probability that the lifetime of a watch is at least 6 years.

A. 0.44                      B. 0.50                      C. 0.56                      D. 0.61                      E. 0.67

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First we find  $\alpha$

$$\begin{aligned}E[X] &= 8 \\ \frac{\alpha(4)}{\alpha-1} &= 8 \\ \alpha &= 2\end{aligned}$$

Then we need to find  $S_0(6)$

$$\begin{aligned}S_0(x) &= 1 - F_0(x) \\ S_0(x) &= 1 - \left(1 - \left(\frac{4}{x}\right)^2\right) = \left(\frac{4}{x}\right)^2 \\ S_0(6) &= \left(\frac{4}{6}\right)^2 = \boxed{0.444}\end{aligned}$$