The Volatility Smile Ch 18: Patterns of Volatility Change

Overview

- Various heuristic relationships between $\frac{\partial \Sigma}{\partial K}$ and $\frac{\partial \Sigma}{\partial S}$, such as:
  - The sticky strike rule
  - The sticky moneyness rule
  - The sticky delta rule
  - The sticky local volatility model

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- The slope of the volatility smile, $\frac{\partial \Sigma}{\partial K}$, is what is observed from market data

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- Under the presence of negative skew ($\beta > 0$), the ATM implied volatility **decreases** when S **increases**

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- The implied volatility of an option with the same *moneyness* should always be the same, regardless of the level of the stock price
- When $\beta > 0$ (negative skew), it follows that $\frac{\partial \Sigma}{\partial S} > 0$
- Thus, under the sticky-moneyness rule, the correct hedge ratio for a standard option will be **greater** than the BSM delta

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A linear approximation of the sticky-delta rule is:

$$\Sigma(S,K) = \Sigma_0 - \beta \frac{\ln\left(\frac{K}{S}\right)}{\Sigma_{ATM}(S)\sqrt{\tau}}$$

Sticky Local Volatility Model

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- Thus, an increase in the index has the same impact on the implied volatility as an equal increase in the strike
- This is the opposite of what was assumed for the sticky moneyness and sticky delta rules

Summary

Heuristic	**Linear Approximation of Σ**	**Δ vs. Δ_{BSM}**
Sticky strike	$\Sigma(S,K) = \Sigma_0 - \beta(K - S_0)$	$\Delta = \Delta_{BSM}$
Sticky moneyness	$\Sigma(S,K) = \Sigma_0 - \beta(K - S)$	$\Delta > \Delta_{BSM}$
Sticky delta	$\Sigma(S,K) = \Sigma_0 - \beta\frac{\ln\left(\frac{K}{S}\right)}{\Sigma_{ATM}(S)\sqrt{\tau}}$	$\Delta > \Delta_{BSM}$
Local volatility	$\Sigma(S,K) = \Sigma_0 + 2\beta S_0 - \beta(S + K)$	$\Delta < \Delta_{BSM}$