This is an excerpt from the full handout file for sample purposes

Experience Study Calculations

Reflecting Withdrawal (Lapse) Rates

If lives can withdraw due to lapse, etc.:

$$\ell_{x+1} = \ell_x - d_x - w_x$$

Using the **Balducci Hypothesis**, we can calculate $q_x = d_x / E_x$

$$E_x = \begin{cases} \ell_x - \sum_{i=1}^{w_x}(1 - t_i) = (\ell_x - w_x) + \sum_{i=1}^{w_x} t_i & \text{for individual calculations} \\ \ell_x - \frac{1}{2} w_x & \text{for grouped calculations} \end{cases}$$

t_i = fraction of the year when each withdrawal i occurs

Balducci assumes mortality decreases over the course of the year

- Not realistic, but not a problem if w_x's are small

Individual Exposure Calculation (Cohort Study)

Assume a 4-year LY study on 3 lives

		Exposure for age x			
Life	Situation	65	66	67	68
A	Survives to 69th birthday	1.0000	1.0000	1.0000	1.0000
B	Dies between 66th and 67th birthdays	1.0000	1.0000		
C	Lapses 110 days after 67th birthday	1.0000	1.0000	0.3014	
	Annual Exposure (E_x)	3.0000	3.0000	1.3014	1.0000

- Deaths get a full year of exposure (Life B)
- Withdrawal exposure for Life C in year of age 66 = 110/365
- Annual exposure by year:

$$\ell_{65} = \ell_{66} = 3 = E_{65} = E_{66} \text{ since no } d\text{'s or } w\text{'s}$$
$$\ell_{67} = 3 - 1 - 0 = 2 \text{ since 1 death occurred}$$
$$E_{67} = \ell_{67} - 1(1 - 0.3014) = 2 - 0.6986 = 1.3014$$
$$\ell_{68} = 2 - 0 - 1 = 1 = E_{68} \text{ since no other decrements}$$

Multi-Year Studies

Exposure-weighted average mortality rate over an N-year period starting at age x:

$$q = \frac{\sum_{t=0}^{N} E_{x+t} q_{x+t}}{\sum_{t=0}^{N} E_{x+t}}$$
$$= \frac{\sum_{t=0}^{N} d_{x+t}}{\sum_{t=0}^{N} E_{x+t}}$$

From the previous slide, the probability of someone age 65 dying before age 69:

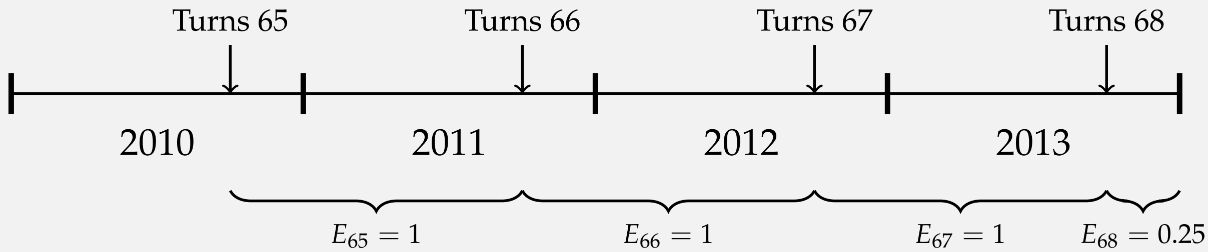
$$_4q_{65} = \frac{1}{3 + 3 + 1.3014 + 1} = 12\%$$

Not very credible since we only had 3 lives!

Period Study: Individual Exposure Example 1

Assume the following:

- Study runs from 1/1/2010 to 12/31/2013
- Minimum age = 65
- Policyholder turns 65 on 10/1/2010 $\approx$ 75% into 2010
- Policyholder survives entire study

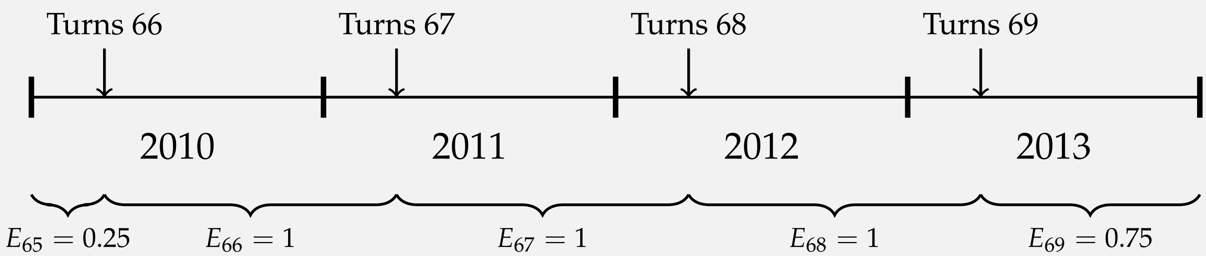


Total life years exposed = 3.25

Period Study: Individual Exposure Example 2

Assume the following:

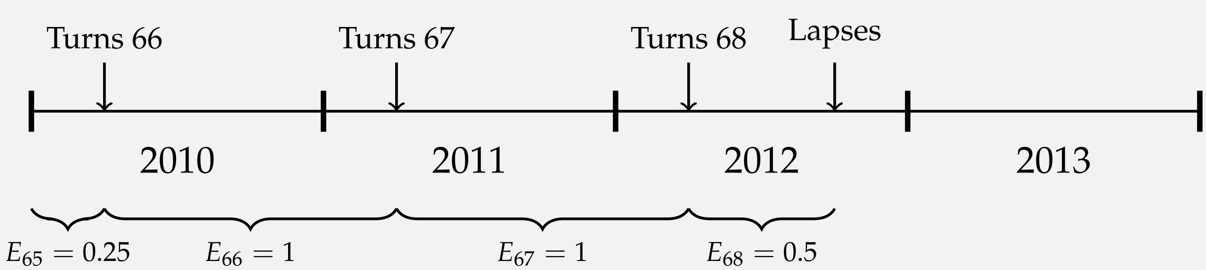
- Study runs from 1/1/2010 to 12/31/2013
- Minimum age = 65
- Policyholder turned 65 on 4/1/2009 $\approx$ 25% into <u>2009</u>
- Policyholder survives entire study



Total life years exposed = 4

Period Study: Individual Exposure Example 3

Assume Example 2 except that the policyholder lapses on 10/1/2012



If the policyholder had <u>died</u> on 10/1/2012, $E_{68} = 1$ (always give a full year)

Review of Individual Exposure Period Studies

- Exposure depends on when the min age is met (before or after study start)
- First or last year will have partial exposures for each life
  - Exceptions: January 1 birthdays or deaths in final partial year
- Deaths and withdrawals are treated the same as a cohort study
- Aggregate mortality rate is calculated the same as before

$$q = \frac{\sum_{t=0}^{N} d_{x+t}}{\sum_{t=0}^{N} E_{x+t}}$$