Contents

Mildenhall Ch 3: Risk and Risk Measures

1.1 Introduction

The term **Risk** refers to the uncertainty of achieving objectives

- Note that insurers can also use the term "risk" to refer to a peril or a account

The text distinguishes between various types of risk:

- **Pure risk/ Insurance risk**: possible bad outcome, with no potential of a good outcome
  - The loss on an insurance policy is an example of a pure risk
- **Speculative risk/ Asset risk**: this could have a good or bad outcome
  - UW profit from insurance exposure (premium − loss & expense) can be considered to be a speculative risk

Financial risk refers to the uncertainty of **financial** outcomes.

- Financial risk can involve uncertainty in timing, amount, or both

Insureds are exposed to financial risk: they do not know if/ when a loss can occur, nor its severity. Insurance can reduce this financial risk to the insured by specifying payment dates, or covering a portion of the loss.

Finally, it compares process risk and parameter risk:

- **Process risk**: the uncertainty that arises from a random process
- **Parameter risk**: the uncertainty in estimating the parameters

1.2 Diversifiable Risk

Insurance benefits from the concept of **diversification**, where adding an independent policy to a portfolio increases the aggregate risk by less than the risk of the standalone policy.

- The insurer needs to understand if a risk is diversifiable (aka **idiosyncratic**)
- **Diversification benefit**: adding a unit to a portfolio increases the risk of the portfolio less than the risk of the standalone unit
- Risk that is nondiversifiable is said to be **systematic**. This typically means that:
  - there is a common underlying cause or some other factor causing dependence among multiple units (e.g. catastrophes that impact multiple units), or

  - there is a single unit that has a large influence on the total loss (e.g. a line of business that has huge losses relative to the other lines)

We can look at diversification and systematic risk in the context of simulations: in each iteration of a simulation,
- If variables are regenerated separately for each unit, they should typically diversify
- If the variable is shared between units, this will introduce systematic risk (each unit will be impacted by a single event)

Note that systematic risk is different to **systemic risk**, which is where an individual event can cause a chain reaction of additional events. The financial market is typically impacted by systemic risk as it consists of many interacting parties, and therefore a single event can impact many of the participants.
- Regulators have identified **Systemically important financial institutions** (SIFIs), which are firms that are considered to generate systemic risk, and are considered to be "too big to fail"
- Even though P&C insurers are not typically considered to be SIFIs (because they have a combination of a large amounts of liquid assets and illiquid liabilities), certain insurers were classified as SIFIs following the Global Financial Crisis of 2008

Catastrophes are a good example to illustrate the difference between systematic and systemic risk:
- Catastrophes are considered to be systematic risk since many insureds are impacted.
- Catastrophes are not considered to be systemic, as losses are generally not amplified by a chain reaction of events.

1.3 Representing Risky Outcomes

The main reason that the authors had introduced the concept of risk is as background for the next topic: "risky outcomes" (e.g. loss amount).

There are several different ways to label a risky outcome:
- Explicit: describe the factors that cause it
- Implicit: identify the outcome by its value
- Dual implicit: identify it by the probability of observing no larger value

These are all described below.

1.3.1 Explicit Representation

An **"explicit representation"** identifies an event by providing specific details about it. For example, the following factors can be used for an auto loss:
- Policy number
- VIN
- Date & time of loss
- GPS location of accident

Explicit representation can distinguish between different events based on these factors, even if they have the same loss dollars.

Let Ω represent the universe of different combinations of selected variables (the **"Sample space"**). Each specific combination within the sample space is represented as $\omega \in \Omega$. Note that there may be other variables in the dataset that are not vital to identify the event (for example, loss adjuster name). These ancillary fields are not included in ω, but rather are considered to be "functions" of ω, and can be linked to the event if necessary.

Lloyd's of London uses Explicit representation to aggregate catastrophe risk: each member needs to report estimated losses from 16 hypothetical events: "Realistic Disaster Scenarios" (RDS), including Miami Windstorm and Japan Typhoon, and these losses are aggregated. *This will be discussed further in the text.*

The main advantage of this approach is the ability to link outcomes across a book of business, which helps in the modeling of dependence risk. However, if there are too many possible events, or if the events impact only a small portion of the portfolio, it likely does not make sense to use explicit representation as there isn't sufficient benefit relative to the complexity of applying this practice.

1.3.2 Implicit Representation

An **"implicit representation"** defines an event by its value. This may be appropriate to use if the loss outcome is what is important, as opposed to the cause of loss.

The Sample space in this case is any value: ($-\infty$, ∞)

There are a few disadvantages of this approach:

- It is difficult to aggregate events as there is no way to link the different outcomes of an event
- It is difficult to specify dependence
- We can not distinguish between different events that have the same outcome

As an example, assume that the cat model output for an insurer covers both the homeowners and commercial lines of business. The spreadsheet with the output for Homeowners has the following columns:

- Hurricane/ Earthquake flag
- Homeowners loss

This data can be used to derive Homeowners loss distributions by peril as well as in total. However it can not be used to link a Homeowners loss to a Commercial loss from the same event, as we only have the value and therefore there is no way to connect the two.

1.3.3 Dual Implicit Representation

"Dual implicit representation" is even more of a simplification than Implicit representation. It could be used if we don't care about the loss size, but rather just the rank of the outcomes.

Here an outcome $X = x$ is identified by its nonexceedance probability:

$$p = F(x) = Pr(X \leq x)$$

An equivalent way to look at this is with the exceedance probability:

$$s = S(x) = Pr(X > x)$$

The Sample space in this case is: (0, 1)

We can also determine x from a given p value by using the inverse of the distribution function (excel has functions to accomplish this such as BETA.INV, GAMMA.INV, LONGNORM.INV, NORM.INV, etc).

Example:
Assume that a normal distribution has a mean of 0 and SD of 1. If the p value is 0.7, calculate x

In Excel, enter NORM.INV(0.7,0,1)
x = 0.524

Examples of dual implicit representation include:

- Investors assess bonds based on their estimated probability of default
- Results of catastrophe models are often summarized by the exceedance probability

One of the advantages of this representation is that it is easy to make comparisons (especially of different events), as regardless of the possible value of X, F(X) always lies between 0 and 1.

Disadvantages include:

- Similar to implicit representation, it is difficult to aggregate
- In practice, events are often compared to an unspecified reference portfolio. For example some news reports described Hurricane Katrina as a 1 in 25 year event (which was comparing to all hurricanes landing in the U.S.), whereas other reports described it as a 1 in 300 year event (comparing to storms that were in the same area). It therefore can be misleading

The text focuses on this metric.

1.4 Conclusion on Representations

The representations discussed previously are all used in insurance. For example:

- Explicit: specific claim file
- Implicit: loss amount from the claim (ignores other details)
- Dual implicit: nonexceedance probability of the loss

Risk Measures

2.1 Intro

A **risk preference** is a way of measuring preferences between different risks. It is denoted by the symbol $\succeq$.

- X $\succeq$ Y: risk X is preferred to Y
- If both X $\succeq$ Y and Y $\succeq$ X: we are neutral between X & Y

A risk preference for insurance loss outcomes needs to have the following properties:

- **Complete (COM)**: for any pair of losses X & Y, we can conclude that X $\succeq$ Y, or Y $\succeq$ X, or both
  - In this case, we are able to perform comparisons between any two pairs
- **Transitive (TR)**: If X $\succeq$ Y and Y $\succeq$ Z, then X $\succeq$ Z
  - This means that the risk preference is logically consistent
- **Monotonic (MONO)**: If X $\leq$ Y for all outcomes, then X $\succeq$ Y

A risk measure is a way to quantify risk preferences (via a single number). In the text, $\rho(X)$ is a risk measure that is based on loss X

Lower risk measures are preferable for the insurer:

$$X \succeq Y \Leftrightarrow \rho(X) \leq \rho(Y)$$

Examples of a "risk measure" include:

- **RBC**: generates target capital need by applying factors to certain items from the insurer's financials; where the factors are based on the level of risk
- **Classification rating plans**: generates a premium based on the risk characteristics (e.g. building value, location, construction, etc)

Risk measures are impacted by the following factors:

- **Volume**: smaller risks are preferred
- **Volatility**: lower volatility is preferred
- **Tail**: lower tail is preferred
  - Note that this is not necessarily the same as volatility. For example, the chance of winning $1M or $3M has the same variability as the chance of winning $1M or losing $1M; but the tail risk in the latter case is much more severe

2.2 Applications of Risk Measures

Insurance company operations are heavily driven by the following risk measures:

- **Capital risk measure**: generates the capital need to write the desired business, which would be based on the level of risk. Note that this could be applied for a single policy, segment, or whole portfolio.
  - the capital need is based on the amount of assets required to be able to pay the losses at a given level of confidence
  - an alternative way to approach this is to determine the business that the insurer can write based on a hypothetical capital amount
  - the capital risk measure is typically more **sensitive to tail risk** as capital is typically required to support tail events that threaten the insurer's solvency
- **Pricing risk measure**: determines the cost of insurance required to provide investors (in the insurer) a return that is sufficient profit to compensate them for bearing the risk
  - the resulting **margin** (equal to premium − expected loss) needs to be sufficient for the insurer to be able to attract the required capital (from investors)
  - this measure is typically more **sensitive to volatility** as management & investors are concerned about volatility of earnings in addition to solvency
  - Pricing risk measures are also referred to as **premium calculation principles**

We will see each of these types of risk measures used throughout the text.

The risk measures have several uses. The text briefly discusses the following two, which are related to **conservatism**:

- Treat the level of conservatism as an input, and derive the required premium or capital based on this
  - e.g. generate the capital needed so that the insurer is able to pay for 99% of the loss outcomes
  - **Stress tests** can also be used as the risk measure. For example, Llyods uses the Realistic Disaster Scenarios (described elsewhere) as a risk measure to set capital requirements: It could set capital equal to the worst outcome. Since this is fairly intuitive, it helps communicate its capacity to a nontechnical audience.
- Use the actual price or capital requirement of the insurer to back into the implied level of conservatism
  - e.g. based on the available capital, the insurer is able to pay for x% of the loss outcomes

2.3 Possible Functional Forms for Risk Measures

There are many different functional forms that we can use to create risk measures.

One of the most basic functional forms for a loss X is the mean of X:

$$E[X] = \int_{\Omega} X(\omega)Pr(d\omega) = \int xdF(x)$$

For example, simply set the premium equal to the expected losses.

In addition to this, there are **risk measures that are different versions of the expected value**. These adjustments better reflect the insurer's preferences (such as risk tolerance). They incorporate the variable for the state of the world, ω, that was introduced earlier.

$$\int_{\Omega} g(X(\omega), \omega)Pr(d\omega)$$

- Here the outcome value is adjusted by a factor that depends on both the outcome value & the sample point ω. This sample point can represent the "state of the world". This means that two identical loss amounts could be treated differently in the risk measure if they were generated by different events. For example, losses from certain events that shareholders are more concerned about will receive higher weight, by increasing $g()$.

$$\int_{\Omega} X(\omega)Pr^*(d\omega)$$

- Here the probabilities are adjusted. For example, this adjustment can be used to make specific events possible or impossible. In the latter case, the insurer is essentially removing the impact of events that it is not concerned about.

$$\int_{\Omega} X(\omega)p(\omega)Pr(d\omega)$$

- This is similar to the previous form, but here the probabilities are scaled by a factor $p(\omega)$.
  - Again the insurer can increase (decrease) the weight to events/ scenarios that it is more (less) concerned about
  - Events that were previously impossible would remain impossible under this adjustment.

$$h(\int_{\Omega} g(X(\omega))Pr(d\omega))$$

- Here the adjustment is actually a function of the loss as opposed to ω. Essentially what this function is doing is first calculating the expected value of a transformed loss $\int_{\Omega} g(X(\omega))$, and then applying a transformation to this result (the h function).
  The authors note that the standard deviation has this form, where $h(x) = x^{0.5}$ and $g(x) = (x - \mu)^2$

$$\int_0^\infty \mu(x) dF_X(x)$$

- This does not adjust the probabilities, but simply adjusts the loss outcome independently of the event ω, using a utility function $\mu(x)$ which is not discussed in sufficient detail in the syllabus readings.

$$\int_0^\infty x g'(S_X(x)) dF_X(x)$$

- Here the probabilities are adjusted based on the rank of the loss (survival function of the loss). This leads to "**Spectral Risk Measures**", which are discussed later.

$$\int_0^\infty \mu(x) g'(S_X(x)) dF_X(x)$$

- This is a combination of the prior two approaches: both the loss outcome and the probability are adjusted

Note:

- The authors believe that it is preferable to adjust probabilities instead of outcomes
- In the final 3 methods, the result depends on the distribution of X as opposed to its magnitude. We say that risk measures with this property are **law invariant**.
- It is important that the measure(s) selected are appropriate for the intended use. The user also has the option to generate multiple different risk measures and use a weighted average (or the maximum) of the results.
- Once the risk measure has been created, we can perform additional functions on the result. For example, we can gross up the result to account for operational and unmodeled risk (*discussed elsewhere in the syllabus*).

