



THE INFINITE ACTUARY'S
DETAILED STUDY MANUAL FOR THE

CP 312 Exam

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2026

About This Study Manual

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This detailed study manual is just one component of our CP 312 course, which also includes comprehensive video lessons, handouts, practice problems, commentary on prior SOA exams, flashcards, and more.

The manual follows the same structure as the online course—organized in a logical learning sequence:

- A. Cash Flow Models
- B. Supplementary Models
- C. Model Governance

We recommend downloading each section's manual before starting its corresponding lessons on the TIA platform — especially if you're using the Today view — so you can reference it as you move through the material.

This manual is designed to help you work through the source readings more efficiently. Some readings are dense enough that you may wonder if the authors were being paid by the word. Do not fret - this is exactly why we built this study manual. Don't worry if every concept doesn't 'click' on the first attempt. Move forward, and return to difficult topics during your second pass.

How to Use This Study Manual With the Online Course

This manual serves as your primary written foundation and reference throughout the course. It is intentionally more detailed than the condensed outlines or flashcards, which are designed for memorization and last-stage review. The focus here is **readability** and **conceptual understanding**, not memorization.

If possible, plan for a 3-4 month study window. Spend the first half learning concepts—without stressing about memorizing details. The final 1-2 months should shift toward review, memorization, and practice.

Each reading begins with an 'Overview of This Reading' section summarizing the major topics and most testable ideas. Review these overviews before and after working through each chapter. Some readings will feel intuitive while others will require patience, caffeine, and possibly a small nap. After reading the detailed manual for a section, watch the video lesson to reinforce key ideas and see additional examples.

Good Luck!

As you progress through the material, keep two principles in mind:

1. Keep moving forward - don't get bogged down!
2. You are not trying to become an expert in the material. You are trying to identify and master testable material sufficiently to pass the exam.

With steady, consistent study, you'll be surprised how much you can retain — *especially if you focus on concepts early and leave memorization for later*. The condensed handouts and flashcards will guide your memorization during the final phase of your preparation.

Finally, remember that one of the most important features of our online course is customer support. Our course forum is the best place to post questions about the material since other students can chime in and learn from the responses as well.

Looking forward to embarking on this journey with you!

Your Instructor,

Shiv Morjaria, FSA, CFA

Change Documentation

We maintain a detailed revision log in the 'Study Schedule' spreadsheet posted in the Introduction section of the online course. Whenever we make any corrections or post new content, the affected parts of the course are listed there.

Check this revision history periodically leading up to the exam. Each PDF includes a version number in the file name so you can confirm you have the most recent edition.

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Section A.1.

Foundations of Actuarial Modeling

LTAM Chapter 1: Introduction to Modeling

Source Author: Tim Cardinal

Overview

Cash flow models are the cornerstone of modern business and risk management, particularly within the insurance industry. These models enable businesses to perform many core functions and develop a deeper understanding of the opportunities and risks embedded in their financial statements. In this reading, we introduce the modeling process and lay the groundwork for more detailed discussions throughout the remainder of the course.

Key topics for the exam include:

- Model Fundamentals
- Structuring Models for Business
- Model Attributes
- Model Considerations and Efficiency

To understand these diverse applications, we must first establish a clear definition of what constitutes an actuarial model.

Model Fundamentals

Within the actuarial space, there are many areas where models are utilized including:

- Pricing and product design
- Financial reporting and analysis
- Business planning and forecasting
- Managing risks, including capital, liquidity, and collateral requirements
- Regulatory and accounting compliance
 - Reserves and capital ensure insurers can meet their obligations and maintain financial stability
 - The emergence of insurance earnings is heavily dependent on these items
 - Therefore, correctly using models to project and understand drivers of these items is critical to success

These models are often used to project items of interest and allow users to draw conclusions. In line with Actuarial Standard of Practice (ASOP) 56, every model can be understood as having three core components:

1. **Information Inputs:** The data, assumptions, and parameters that serve as the raw materials for the model

2. **Processing:** The *calculation engine* that applies mathematical formulae and algorithms to transform the inputs
3. **Results:** The outputs of the calculation, which are transformed into useful business information or intelligence for decision-making

In practice, actuarial models follow a robust framework with the following items:

1. Databases
 - Used to store inputs like parameters, assumptions, scenarios, policyholder data, etc.
 - Used to store outputs such as financial results, pricing metrics, risk management projections, etc.
2. Liability projections (discussed further below)
3. Asset projections
 - Inputs include rates, defaults, prepayments, spreads, etc.
 - Models are used to project undecrement and decremented cash flows, book values, and market values
 - These are used to determine the desired financial output and analysis
 - Since assets and liabilities are interdependent, the interaction between them needs to be modeled as well
 - This can be done *explicitly* or *iteratively* (discussed later in the reading)
4. Valuation models
 - Project the resulting undecrement cash flows, decremented cash flows, and future reserve/capital levels
 - These results are used by the valuation team to analyze experience vs. actual variances, trends, and drivers of value

The remainder of this reading focuses on the actuarial liability models which can be distilled into three basic, interconnected building blocks:

1. **Building Block 1: Modeling Rates and Decrements**

- This block models the rates and decrements used to model populations, states, and the transitions between them
- Examples include the application of mortality, morbidity, benefit utilization, surrenders, and lapse rates
- These are applied to the population in force to understand how the population will evolve over time

2. **Building Block 2: Undecrement Flows**

- This block models cash flows and values from the *policyholders' perspective*, assuming the policies remain in force throughout the policy period

- The cash flows are conditional upon the policyholder being in force at the beginning of each model time-step
- All premiums and benefits are modeled within this building block

3. Building Block 3: Decrementing Flows

- This block models the *insurer's perspective*, which reflects how many policies are actually in force at each projection date
- The decremented cash flow is simply the undecmented cash flow adjusted for the number of policies that remain in force

Understanding these fundamentals about how models are used at insurance companies provides the foundation for the remainder of this course.

Structuring Models for Business

Insurers do not manage individual policies in isolation and, instead, aggregate results to reflect the organizational hierarchy used to manage, operate, and analyze the business.

Model results can be aggregated at the following levels:

- **Lines of Business (LOBs)/Business Segments**
 - Broad groupings of products and markets served
 - E.g. international vs. domestic or property & casualty vs. life & annuity lines of business
- **Blocks of Business/Portfolios**
 - Subsets of an LOB, which are typically determined by specific markets and product types
 - E.g. whole life, universal life, indexed universal life all within the Life division
- **Cohorts:**
 - Subdivisions of a block, often representing a group of policies that share a common characteristic
 - E.g. whole life policies split by issue year
- **Seriatim:**
 - Each calculation cell represents a single liability policy or asset and the entire business is modeled one policy at a time
 - Certain regulatory or accounting rules require seriatim calculations
 - Results from seriatim models can be aggregated to the desired level of output

Depending on the context, insurers will make decisions at different levels of the organization and the model's calculation engine should reflect the applicable level of aggregation and management.

Model Attributes

Actuarial model design is an exercise in professional judgment and balancing trade-offs. A range of qualitative attributes must be considered during the design process including:

| Attribute | Description |
|--------------------|---|
| Usefulness | - The degree to which a model serves the user's purpose - Encompasses functionality, ease of use, and effort required to operate it |
| Complexity | - Refers to the simplicity or complexity of the model's structure - Includes model's architecture, number of parts, and relationships between these parts |
| Fidelity | - The degree to which a model accurately reproduces the item it is meant to represent |
| Accuracy | - The degree to which a model's outputs match the actual values being modeled |
| Precision | - The ability of a measurement to be consistently reproduced - Refers to the level of detail the model is able to reliably produce |
| Speed | - The length of time it takes for a model to run from start to finish - Companies often need to weigh speed against usefulness - Actuaries should also consider the speed of building, validating, implementing, and maintaining models |
| Granularity | - The level of detail built into a model or process - Reflects the number of factors being projected and the number of variables used - Higher granularity often comes at the cost of lower speed or simplicity |
| Robustness | - The degree to which a model effectively supports a wide range of purposes and/or products |
| Flexibility | - The degree and ease with which a model and its components can be adapted or changed |
| Stability | - The model's ability to produce consistent and reliable results over time - An unstable or fragile model often breaks from operations and maintenance |

Actuaries must thoughtfully balance each of these attributes when building models to ensure they are both insightful and manageable. Actuaries also must avoid the 'over-fitting' trap where a model looks good on paper but does not have real explanatory power.

Model Considerations and Efficiency

Actuaries must always remember that a model is simply a means to an end and ultimately only useful if it helps us understand the underlying business and drivers of value. The following items are important to consider during the model design process:

- Models must strike a balance between simplicity and sophistication such that they are practical to use but still provide useful insights
- Since the world is constantly changing, modeling should be viewed as a continuous, iterative process

- Modeling variable interdependencies makes models exponentially more complex but more reflective of reality
 - E.g. modeling dynamic policyholder behavior that varies based on projected market conditions
 - Actuaries must ensure they reflect the *principle of no foresight*
 - This means at each projected model timestep, only the conditions and events that have happened *up to that point in time* can be used to project decision-making behavior
 - Models can distinguish between *proactive actions* and *reactions* to the projected conditions

Actuaries should also ensure they are balancing these considerations against *efficiency* which includes an assessment of:

- Computing power utilization
 - Understanding computational capacity and efficiency (e.g. splitting tasks across processors)
 - Optimizing model speed and calculations using efficient coding practices
 - Using *proxy models* where appropriate
- Scalability of the models
 - Modern business needs require models to perform large volumes of calculations
 - This includes multiple valuation and accounting bases, new lines of business, and regular sensitivity/scenario testing
 - Models that are more flexible can accommodate future changes without requiring overhauls
- Memory management
 - With distributed processing, it is crucial to manage how data is stored and retrieved during calculations
 - Using best coding practices (e.g. avoiding unnecessary nested loops) ensures memory is being used efficiently
 - Actuaries should design models that balance variables that are calculated on-the-fly vs. stored in memory and retrieved
- Integration with other systems
 - Models that seamlessly integrate with other reporting tools, dashboards, and platforms are preferred
 - This integration speeds up the process and minimizes inconsistencies/human errors
- Modeling asset-liability interdependencies

- Assets and liabilities are intrinsically tied to one another in many ways such as:
 - * Newly invested assets affect portfolio yield which can impact liability crediting rates and account values
 - * Excess liability cash flows creates the need for borrowing cash within the projection leading to a drag on projected net earned rates
 - * Excess liability cash flows may force the sale of assets which impacts future projected book values and investment income
- There are two main approaches actuaries can take for modeling these interactions: *explicitly* or *iteratively*
- Explicit projection involves replicating reality as closely as possible
 - * The projection model runs the liability and asset models in tandem one time-step at a time
 - * This takes into account potential reinvestment, borrowing, and selling activity on the asset side
 - * The downstream impacts on portfolio yield, crediting rates, and policyholder behavior are explicitly captured
- Iterative projections involve running the asset and liability model multiple times
 - * The liability model is run using an assumption about the asset earned rate
 - * The resulting liability cash flows from this run are used as an input for the asset model
 - * The resulting earned rates and asset cash flows are used as inputs for the liability model
 - * This process continues iteratively until the model results between step k and $k - 1$ converge to a specific tolerance level
- The explicit approach is more memory intensive and makes splitting tasks across processors difficult or impossible
- The iterative approach requires a larger volume of less intensive runs making it preferable in practice

In practice, it may be more efficient to utilize a 'proxy/light model' which is a model that is intended to replicate the output of a fully detailed ('heavy') model with faster run times and an acceptable reduction in accuracy. In order to assess and improve the proxy model's accuracy, its output is compared to heavy model's output.

This concludes our introduction to modeling. Most of the concepts introduced here will be discussed in more depth for the remainder of the course.

LTAM Chapter 3: Principle-Based Projections

Source Author: Tim Cardinal

Overview

This chapter develops the mechanics of *gross premium valuation* and extends the framework to principle-based projections using *nested modeling techniques*. The material develops a structured framework for model construction, assumption management, and projection design in order to explain how both current and future valuations are determined.

Key topics for the exam include:

- Gross Premium Valuation Fundamentals
- Nested Methods for Projection

Gross Premium Valuation Fundamentals

A gross premium valuation (GPV) method is used to prospectively determine a liability as the present value of future benefits and expenses less gross premiums:

$$\text{Gross Premium Reserve} = \text{PV}(\text{benefits}) + \text{PV}(\text{expenses}) - \text{PV}(\text{gross premiums})$$

Importantly, in a GPV framework, asset performance is often reflected *implicitly* through the discount rate rather than modeled as *explicit* investment income cash flows within the liability projection.

The GPV calculation can be performed deterministically or stochastically:

1. Deterministic

- Projected using a fixed set of inputs to produce a single, predictable output
- Usually means projecting the model using best estimate assumptions along a single scenario

2. Stochastic

- Incorporate randomness and uncertainty by using a range of possible inputs to produce a range of potential outcomes
- E.g. projecting GPV along 1,000 scenarios to understand the distribution of the liability
- Allows actuaries to set the reserve level at a specified VaR or CTE level
 - VaR (Value at Risk) is a specific percentile on the liability distribution
 - VaR(85) represents the 85th percentile of the liability distribution
 - In a 1,000-scenario simulation VaR(85) corresponds to the 150th largest reserve

- CTE (Conditional Tail Expectation) is the average of the tail beyond a specific percentile
- CTE(85) represents the average of all reserve outcomes beyond the 85th percentile
- In a 1,000-scenario simulation, CTE(85) corresponds to the average of the largest 150 reserve outcomes

Since the GPV is calculated prospectively, the biggest driver of results is the set of assumptions used in the valuation. There are several categories of assumptions actuaries can use for valuation depending on the purpose:

- **Anticipated/Best-estimate Assumptions**
 - These are neutral and not intentionally conservative or aggressive
- **Prudent Assumptions**
 - Adds risk margins to the anticipated assumptions for conservatism
 - Risk margins reflect the uncertainty associated with an assumption
 - Greater uncertainty means larger margins to reflect the cost of bearing risk
 - Margins can be additive or multiplicative and are calibrated so that they increase the GPV liability when applied
- **Regulatory Prescribed Assumptions**
 - Assumptions mandated by regulators usually intended to introduce conservatism
 - These assumptions are used for specific regulatory requirements

Once the projection type and assumptions are settled, the GPV projection typically follows the three liability model building blocks:

1. Discount rates, credited rates and decrements are calculated
2. The policyholder-perspective undecrement cash flows and values are calculated
3. The insurer-perspective decremented cash flows and values are determined using the two items above

Practical Valuation Applications

The source material provides a robust example of a Traditional Life Model employing a GPV calculation. Some commonly used acronyms in the example are:

| Acronym | Definition |
|---------|----------------------------------|
| CV/CSV | Cash (Surrender) Value |
| FMI | Future Mortality Improvement |
| KPI | Key Performance Indicator |
| NAR | Net Amount at Risk |
| EBIT | Earnings before Income taxes |
| GPV | Gross premium valuation |
| MI | Mortality Improvement |
| HMI | Historical Mortality Improvement |
| PH | Policyholder |

I would recommend reviewing the Excel spreadsheet provided by the source author to ensure you understand the calculation mechanics. While full spreadsheet replication is unlikely to be tested, individual components of the logic are clearly within scope. I have also included a simplified calculation to illustrate these concepts in the Excel spreadsheet attached to this lesson.

The key takeaways from the example are:

- Any assumptions being made need to be explicitly stated so the user always understands the limitations of the analysis being performed
- In many principle-based frameworks, income statement projections follow best-estimate assumptions, while reserve calculations incorporate margins or prescribed conservatism
- Margins are designed to increase the GPV liability relative to a best-estimate projection:
 - For life insurance death benefits, margins should increase the mortality rate and decrease mortality improvement
 - For living benefits, margins should decrease the mortality rate and increase mortality improvement
 - For lapse rates, the direction of the margin may vary by projection period in order to reflect the most financially adverse policyholder behavior for the insurer
 - For investment income/assets where applicable, margins should increase defaults and decrease investment spreads
- For the income statement, the following relationships are important to remember:
 - Revenue = Investment income + Premiums
 - Cost = Benefits Paid + Expenses + Increase in reserves
 - Net Income = (Revenue - Cost) × (1 - Tax Rate)
 - Distributable Profit = Net Income + Decrease in capital held

From the last bullet point, we can see that future financial results depend on how reserves and capital evolve over time. Therefore, estimating these values at future dates becomes important which gives rise to the need for *nested models*.

Nested Methods for Projection

Nested methods are required when a projected quantity within a projection must itself be calculated using a full valuation model:

- Essentially this gives rise to a sub-projection within the main projection
- The main projection is referred to as the *outer loop* and the projection embedded within it is known as the *inner loop*
- The outer loop is the primary projection path, typically using best-estimate assumptions
- The inner loop projection is performed at each re-valuation date using the assumptions that correspond to the value being calculated
- For a company looking to calculate its future reserve values:
 - The outer loop is modeled using the same projection principles discussed thus far in the chapter
 - At each valuation date, an inner-loop projection is performed to calculate the reserve level on that date
 - This inner-loop re-valuation begins from the outer-loop projected state as of the valuation date
 - The in-force at the start of this projection reflects the population that has survived up to the valuation date
 - The market conditions at the start of this projection reflects the outer loop state at the valuation date

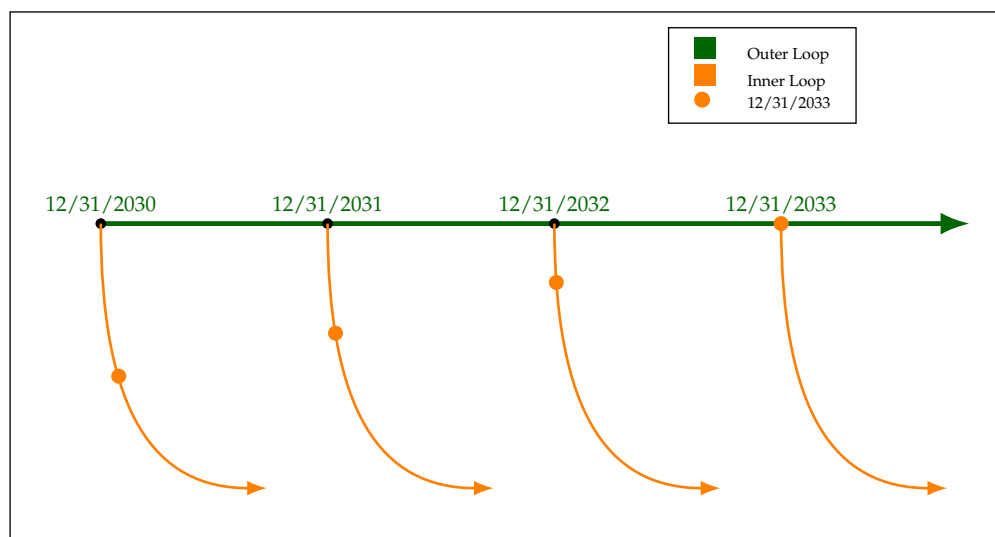
Conceptually, the outer loop projects how the economic and policy environment evolves over time, while the inner loop re-measures the liability given the projected state at a specific future date.

Nested projections help companies answer the question: *what will my reserve and capital levels be in the future?*. A new reserve calculation is performed at each time-step under required standards (usually prescribed assumptions). This has several implications:

- For a given calendar date, the outer-loop projection and various inner-loop projections will generally produce different states because they originate from different projection paths
- As an example, for a projection starting on 12/31/2030, there are multiple 'paths' to the valuation date of 12/31/2033:
 - Outer loop projection period 3; inner loop projection period 0 (starting point of the reserve calculation)
 - Outer loop projection period 2; inner loop projection period 1
 - Outer loop projection period 1; inner loop projection period 2
 - Each of these instances involves a potentially different path with a unique in force profile and market environment

- A full GPV is performed at every future projection date by following the outer loop through that point
- As an example, for a projection starting on 12/31/2030, in order to calculate the reserves over the next three years, the following steps are taken:
 - A full GPV is performed as of 12/31/2030 (no outer loop projection needed)
 - The liability/asset inventory and assumptions are projected along the outer loop for 1 year then a full GPV is performed as of 12/31/2031
 - The liability/asset inventory and assumptions are projected along the outer loop for 2 years then a full GPV is performed as of 12/31/2032
 - The liability/asset inventory and assumptions are projected along the outer loop for 3 years then a full GPV is performed as of 12/31/2033
 - Note that each inner-loop does not restart from the original assumptions but, rather, begins from the projected state generated by the outer-loop

As shown below, a single calendar date can be accessed through multiple paths, each defined by its specific market and in-force experience:



At each year on the outer loop, a full inner loop GPV calculation would be performed in order to calculate the reserve at that date.

The Excel example attached to this lesson provides a simplified example of how the the inner and outer loops interact. The accompanying spreadsheet from the source material¹ provides a more detailed illustration of these mechanics. While the full implementation is unlikely to be examined directly, understanding the structural logic is testable.

Projection Considerations

The following items are important to consider when designing nested stochastic models:

¹ LTAM_II_Example_Ch3.2.xlsm

1. Use Cases

- The primary use case for nested projections is projecting future reserve and capital levels to forecast future net income and distributable earnings
- In these cases, the outer loop follows best estimate assumptions and the inner loop follows conservative or prescribed assumptions
- The outer loop is projected based on the company's internal plan scenario while the inner loop is calculated stochastically and reserves/capital are based on the appropriate CTE level (e.g. CTE 70 or CTE 98)
- Examples of inner loop applications include VM-20, VM-21, RBC C3P2, GAAP, and economic reserves

2. Model Run Time

- Inner-loops are computationally intensive
- This task gets exponentially more difficult if asset projections need to be modeled since asset states depend on cumulative prior interactions with liabilities
- Modelers should apply judgement when deciding how many/which outer loop projection time steps to run

3. Assumption Management

- Modelers must determine which assumptions are re-determined at each projection time steps
- Some assumptions may continue to use the initial valuation date values
- Judgement is required based on materiality and run time

Nested projections therefore represent a structural necessity whenever future reserve and capital levels must be projected under differing valuation bases across time. The biggest thing to remember is that the outer loop represents the anticipated evolution of the world and the inner loop provides a re-measurement of the liability if the world evolves in that way.